1 Problem of the week(s), 30 October-13 November 2000

The following is from the 37th Putnam exam. Students will have an opportunity to present their solution at the next problem session, Monday, 13 November 2000.

1. Let $R$ be the set of points inside and on a convex polygon in the $(x,y)$-plane. Let $\delta(x,y)$ denote the distance from a point $(x,y)$ in the plane to the point in $R$ which is closest to $(x,y)$. Show that there are constants $a$, $b$ and $c$, which are independent of $R$ so that

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\delta(x,y)} \, dx \, dy = a + bL + cA$$

where $L$ is the perimeter of $R$ and $A$ is the area of $A$. Find the values of $a$, $b$ and $c$.

Hints: 1. Consider simple special cases such as when $R$ is a point or a line segment. 2. What do the sets $\delta(x,y) = \text{constant}$ look like?