Instructions: The purpose of this assignment is to develop your ability to formulate and communicate mathematical arguments.

Please write a short essay in response to each of the following questions, following the standard rules for grammar and editing. You should provide well-written, complete answers to each of the questions. It is strongly recommended that you use word processing software (such as Microsoft Word or Libre Office) for this assignment, with hand-drawn graphs and pictures.

When using mathematical symbols in your essay, either use them as part of complete sentences or display them separately from your paragraphs. Your textbook is a good model for this type of writing.

Your complete assignment should:

• have your name and section number on each page,
• be stapled, and
• be neat and legible.

Unreadable work will receive no credit.

Question A: If the function $f$ is continuous on $[-1, 1]$ and $f(-1) = -1$, $f(0) = 1$ and $f(1) = 0$, is it possible that $f$ is one-to-one? If the answer is yes, give an example of a one-to-one function satisfying these conditions. If the answer is no, provide a justification that any function satisfying these conditions cannot be one-to-one.

Question B: Consider the graphs of the two functions $f(x) = 1 - x^2$ for $-1 \leq x \leq 1$ and $g(x) = 4 - (x-4)^2$ for $2 \leq x \leq 6$. These two functions have a common tangent line. Find the slope of that tangent line. Include a graph of the functions and the common tangent line.

Hint: Find the tangent line to $y = f(x)$ that intersects $y = g(x)$ in exactly one point.