Instructions: The purpose of this assignment is to develop your ability to formulate and communicate mathematical arguments.

Please write a short essay in response to each of the following questions, following the standard rules for grammar and editing. You should provide well-written, complete answers to each of the questions. It is strongly recommended that you use word processing software (such as Microsoft Word or Libre Office) for this assignment, with hand-drawn graphs and pictures.

When using mathematical symbols in your essay, either use them as part of complete sentences or display them separately from your paragraphs. Your textbook is a good model for this type of writing.

Your complete assignment should:

- have your name and section number on each page,
- be stapled, and
- be neat and legible.

Unreadable work will receive no credit.

Question A: Let \((a_1, a_2)\) be a point in the plane. Consider the line \(L\) given by the points \((x, x)\) as \(x\) ranges over all values of \(x\). For what value of \(x\) is the point \((x, x)\) closest to \((a_1, a_2)\)? Justify your answer.

Question B: Given \(n\) numbers \(a_1, a_2, \ldots, a_n\), find the value of \(x\) minimizing the sum of squares

\[(x - a_1)^2 + (x - a_2)^2 + \cdots + (x - a_n)^2.

How is this related to Question A?

Question C: This problem deals with using Newton’s method to approximate square roots. Let \(f(x) = x^2 - c\) — thus, the solution to \(f(x) = 0\) is \(x = \sqrt{c}\). Let \(e_n = x_n - \sqrt{c}\) be the error in \(x_n\).

1. Show that \(x_{n+1} = \frac{1}{2}(x_n + c/x_n)\) and \(e_{n+1} = e_n^2 / 2x_n\).
2. Show that if \(x_0 > \sqrt{c}\), then \(x_n > \sqrt{c}\) for all values of \(n\).
3. Show that if \(x_0 > \sqrt{c}\), then \(e_{n+1} \leq e_n^2 / (2\sqrt{c})\).
4. Explain the meaning of these results without using any mathematical symbols. In other words, explain this behavior of \(x_n\) and \(e_n\) conceptually and qualitatively, rather than symbolically and quantitatively.