# 7 Functions Worksheet

**Concepts:**
- The Definition of A Function
- Function Notation
- Piecewise-defined Functions
  - Evaluating Piecewise-defined Functions
  - Sketching the Graph of a Piecewise-defined Functions
- The Domain of a Function
- Graphs of Functions
  - Identifying Graphs of Functions (Vertical Line Test)
  - Interpreting Graphs of Functions
  - Sketching Graphs of Functions
  - Applying Transformations to the Graph of a Function
  - How Does a Graph Transformation Move a Point on a Graph?
- Operations on Functions
- The Domain of a Composition of Functions.
- Average Rates of Change
  - Calculating the Average Rate of Change of a Function on an Interval
  - Secant Lines
  - Difference Quotients
  - Approximating Instantaneous Rates of Change
1. Which of the following tables could describe a function? Explain your answer.

   (a) Input | ♠ | □ | ♦ | ♢ | ♣<br>   Output | ◯ | ◯ | ◯ | ◯ | ◯

   (b) Input | ♦ | ♠ | ♠ | ♠ | ♠<br>   Output | ♠ | ♠ | ♠ | ♠ | ♠

2. Which of the following equations define \( q \) as a function of \( r \)? Which of the following equations define \( r \) as a function of \( q \)?

   (a) \( qr = 2 \)
   (b) \( qr = 0 \)
   (c) \((q + 1)^3 - r^2 = 7r\)
   (d) \(\sqrt{q} + \sqrt{r} - 1 = 0\)
   (e) \(\left(\frac{2 - q}{7}\right)^5 + \left(\frac{2r + 1}{6}\right) = 1\)

3. Let Joni(\(x\)) = \(x^2 + 1\).
   (a) What is Joni(\(a + b\))?
   (b) What is Joni(\(x - 1\))?
4. Let \( g(x) = x^2 + x \).
   
   (a) What is \( g(2x) \) ?
   
   (b) What is \( g(x^2) \) ?
   
   (c) What is \( (g(x))^2 \) ?
   
   (d) What is \( \frac{g(x+h) - g(x)}{h} \) ?

5. Let 
   
   \[ h(x) = \begin{cases} 
   10 & \text{if } x < -4 \\
   x^2 + 10 & \text{if } -4 \leq x \leq 6 \\
   x + 15 & \text{if } x > 6 
   \end{cases} \]

   - Find \( h(5) \).
   - Find \( h(-4) \).
   - Find \( h(-6) \).
   - Find \( h(6) \).
   - Find \( h(10) \).


   If your taxable income on Form 2, line 45 is:

<table>
<thead>
<tr>
<th>More than</th>
<th>But not more than</th>
<th>Then your tax is</th>
<th>Less:</th>
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<tbody>
<tr>
<td>$0</td>
<td>$2,600</td>
<td>1% of your taxable income</td>
<td>($26)</td>
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<td>$2,600</td>
<td>$4,500</td>
<td>2% of your taxable income</td>
<td>($71)</td>
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<tr>
<td>$4,500</td>
<td>$6,900</td>
<td>3% of your taxable income</td>
<td>($140)</td>
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<td>$6,900</td>
<td>$9,300</td>
<td>4% of your taxable income</td>
<td>($233)</td>
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<td>$9,300</td>
<td>$12,000</td>
<td>5% of your taxable income</td>
<td>($353)</td>
</tr>
<tr>
<td>$12,000</td>
<td>$15,400</td>
<td>6% of your taxable income</td>
<td>($492)</td>
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</table>

   They give the following example.

   For example: Taxable income $6,800 \times .03(3\%) = $204. $204 minus $71 = $133 tax

   Use this tax table to write a piecewise-defined function \( \text{MontanaTax}(i) \) where \( i \) is the income on Form 2 line 45 of the Montana State Tax Form, and \( \text{MontanaTax}(i) \) is the amount of tax owed by a resident of Montana.
7. Find the domain of the following functions. Write the domain in interval notation.

(a) \( a(x) = x^5 + 2x^2 - 6 \)
(b) \( b(x) = \frac{x + 1}{x - 5} + \frac{x + 4}{2x + 1} \)
(c) \( c(x) = \sqrt{x + 7} \)
(d) \( d(x) = \sqrt{x + 7} \)
(e) \( e(x) = \frac{1}{\sqrt{10 - x}} \)
(f) \( f(x) = \frac{1}{\sqrt{10 - x}} \)
(g) \( g(x) = \sqrt{x + 7} - \frac{1}{x^2 - 5} \)
(h) \( h(x) = \begin{cases} 
\frac{1}{x} & \text{if } x \leq -2 \\
\frac{1}{x + 3} & \text{if } x > -2 
\end{cases} \)

8. Find the domain and range of each of the following functions.

\[ \begin{array}{cccccccc}
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\end{array} \]
9. The graph of \( y = f(x) \) is shown below.

(a) For what \( x \) values is \( f(x) \geq 0 \)? Write your answer in interval notation.
(b) For what \( x \) values is \( f(x) < 0 \)? Write your answer in interval notation.
(c) For what \( x \) values is \( f(x) \leq -1 \)? Write your answer in interval notation.
(d) What is \( \frac{f(3) - f(2)}{2f(-6)} \)?

10. For each of the graphs below, answer the following questions:

- Is \( y \) a function of \( x \)?
- Is \( x \) a function of \( y \)?
11. Suppose that the graph of \( f \) contains the point \((-1, 5)\). Find a point that must be on the graph of \( g \).

(a) \( g(x) = \frac{3f(x) + 1}{2} \)

(b) \( g(x) = f \left( \frac{x - 1}{3} \right) \)

(c) \( g(x) = 2f(3(x + 1)) - 6 \)

(d) \( g(x) = 2(f(3(x + 1)) - 6) \)

12. Let \( f(x) = x^2 \). Write \( g(x) \) in terms of \( f(x) \) and explain how you would transform the graph of \( f \) to draw the graph of \( g \). Sketch the graph of \( g \).

(a) \( g(x) = (x + 1)^2 - 3 \)

(b) \( g(x) = 2(x^2 - 5) \)

(c) \( g(x) = 2x^2 + 4x - 1 \) \( \text{(HINT: Complete the square.)} \)
13. The graph of $y = f(x)$ is the solid graph and the graph of $y = g(x)$ is the dashed graph. Find a formula for $g(x)$ in terms of $f(x)$.

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15. The graph of $y = f(x)$ is the solid graph and the graph of $y = g(x)$ is the dashed graph. Find a formula for $g(x)$ in terms of $f(x)$. 
16. Let \( f(x) = x^2 + 3 \) and \( g(x) = 2 - x \).
   (a) Find \( f(g(x)) \).
   (b) Find \( g(f(x)) \).
   (c) Find \( f(f(x)) \).
   (d) Find \( g(g(x)) \).
   (e) Find \( g(g(g(x)))) \).

17. Let \( f(x) = \frac{x}{\sqrt{x+1}} \) and \( g(x) = 2x + 5 \).
   (a) Find \( f(g(x)) \).
   (b) Find the domain \( f(g(x)) \).
   (c) Find \( g(f(x)) \).
   (d) Find the domain \( g(f(x)) \).

18. Let \( f(x) = \sqrt{x} \) and \( g(x) = x^2 \).
   (a) Find \( f(g(x)) \).
   (b) Find the domain \( f(g(x)) \).
   (c) Find \( g(f(x)) \).
   (d) Find the domain \( g(f(x)) \).

19. Write \( h(x) \) as a composition of three simpler functions. (**HINT:** Think of placing \( x \) in a box. What happens first? second? etc.? There may be more than one correct answer.)
   (a) \( h(x) = \sqrt{x^3 + 5} \)
   (b) \( h(x) = \frac{3}{x^5 - 7} \)
   (c) \( h(x) = 3(x + 5)^2 \)
   (d) \( h(x) = (3x + 5)^2 \)

20. Write \( g(x) = 2(f(3(x + 1)) - 6) \) as a composition of five simpler functions (Hint: One of these functions should be \( f \). Draw your composition as a machine diagram. Where is \( f \) in the machine order? How does this help illustrate how to transform the graph of \( f(x) \) to draw the graph of \( g(x) \)?)
21. You have a 15% off coupon from the manufacturer good for the purchase of a new cell phone. Your cell provider is also offering a $10 discount on any new phone. You make two trips to cell phone stores to look at various phones. On your first trip, you speak with Mandy. Mandy tells you that you can take advantage of both the coupon and the discount. She will apply the discount and then apply the coupon to the reduced price. On your second trip, you talk to Kelly. She also says that you can take advantage of both deals, but she tells you that she will apply the coupon and then apply the discount.

Let $x$ represent the original sticker price of the cell phone.

(a) Suppose that only the 15% discount applies. Find a function $f$ that models the purchase price of the cell phone as a function of the sticker price $x$.

(b) Suppose that only the $5 coupon applies. Find a function $g$ that models the purchase price of the cell phone as a function of the sticker price $x$.

(c) If you can take advantage of both deals, then the price you will pay is either $f(g(x))$ or $g(f(x))$, depending on the order in which the coupon and the discount are applied to the price. Find $f(g(x))$ and $g(f(x))$.

(d) The price that Mandy is offering you is modeled by ___________.

(e) The price that Kelly is offering you is modeled by ___________.

22. You have a 20% off coupon from the manufacturer good for the purchase of a new cell phone. Your cell provider is also offering a 10% discount on any new phone. You make two trips to cell phone stores to look at various phones. On your first trip, you speak with Jamie. Jamie tells you that you can take advantage of both the coupon and the discount. She will apply the discount and then apply the coupon to the reduced price. On your second trip, you talk to Mia. She also says that you can take advantage of both deals, but she tells you that she will apply the coupon and then apply the discount.

Let $x$ represent the original sticker price of the cell phone.

(a) Suppose that only the 20% discount applies. Find a function $f$ that models the purchase price of the cell phone as a function of the sticker price $x$.

(b) Suppose that only the 10% coupon applies. Find a function $g$ that models the purchase price of the cell phone as a function of the sticker price $x$.

(c) If you can take advantage of both deals, then the price you will pay is either $f(g(x))$ or $g(f(x))$, depending on the order in which the coupon and the discount are applied to the price. Find $f(g(x))$ and $g(f(x))$.

(d) The price that Jamie is offering you is modeled by ___________.

(e) The price that Mia is offering you is modeled by ___________.

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23. Let \( f(x) = |x - 3| \).

(a) Find the average rate of change of \( f(x) \) with respect to \( x \) as \( x \) changes from \( x = 5 \) to \( x = 7 \). Sketch a graph that illustrates the geometric interpretation of this average rate of change.

(b) Find the average rate of change of \( f(x) \) with respect to \( x \) as \( x \) changes from \( x = -4 \) to \( x = -1 \). Sketch a graph that illustrates the geometric interpretation of this average rate of change.

(c) Find the average rate of change of \( f(x) \) with respect to \( x \) as \( x \) changes from \( x = 2 \) to \( x = 6 \). Sketch a graph that illustrates the geometric interpretation of this average rate of change.

24. A pebble is dropped into a pond creating circular ripples. The radius of the outer ripple is expanding at a rate of 2 feet per second.

(a) What is the average rate of change of the radius of the outer ripple with respect to time? Be sure to include correct units.

(b) What is the average rate of change of the circumference of the outer ripple with respect to the radius? Be sure to include correct units.

(c) What is the average rate of change of the circumference of the outer ripple with respect to time? Be sure to include correct units.

(d) What is the average rate of change of the area enclosed by the outer ripple with respect to the radius? Be sure to include correct units.

(e) What is the average rate of change of the area enclosed by the outer ripple with respect to time? Be sure to include correct units.

25. Find the difference quotient of the function. Simplify your answer.

(a) \( f(x) = x^2 - 3x \)

(b) \( g(x) = x^3 \)

(c) \( h(x) = \sqrt{x + 1} \)

26. Which of the following functions are one-to-one?

(a) The function that maps a word to the number of letters in the word.

(b) The function that maps the year of a Summer Olympics to the winner of the marathon in that Olympics.

(c) The function that maps a U.S. state to its two letter postal code.

(d) The function that maps a person to his or her name.

(e) The function that maps a person to his or her address.
27. The graph of a one-to-one function is shown below. (How do you know that this is a one-to-one function?) Sketch the graph of its inverse.

28. Which of the following graphs display a one-to-one function? If the graph displays a one-to-one function, sketch its inverse.
29. Find the inverse of the one-to-one functions below. Find the domains and ranges of the function and its inverse.

(a) \( f(x) = \frac{2 - x^3}{7} \)
(b) \( g(x) = \frac{x + 7}{x + 5} \)
(c) \( h(x) = \sqrt{x^5 - 2} \)

30. The following functions are not one-to-one. Restrict the domain so that the function is one-to-one. Remember that the range of the function with the restricted domain should be the same as the range of the original function. Find the inverse of the function with the restricted domain.

(a) \( f(x) = (x + 2)^2 \)
(b) \( g(x) = 3x^2 + 6x + 5 \) (You should look at the graph of this function. Draw the graph without using your calculator. Completing the square will help.)
(c) \( h(x) = 2|x - 5| - 3 \) (Draw this graph without using your calculator.)

31. Use the Round Trip Theorem to determine if the pair of functions are inverses of each other.

(a) \( f(x) = \frac{1}{x} \) and \( g(x) = \frac{1}{x} \)
(b) \( h(x) = \frac{3x + 2}{7} \) and \( j(x) = \frac{7x - 2}{3} \)
(c) \( k(x) = \sqrt{3x - 3} \) and \( m(x) = x^3 + 27 \)