Worksheet # 22: Newton’s Method and Antiderivatives

1. Use Newton’s method to find an approximation to $\sqrt[3]{2}$. You may do this by finding a solution of $x^3 - 2 = 0$.

2. Use Newton’s method to approximate the critical points of the function $f(x) = x^5 - 7x^2 + x$.

3. Let $f(x) = \frac{x}{1 + x^2}$.
   (a) Solve $f(x) = 0$ without using Newton’s method.
   (b) Use Newton’s method to solve $f(x) = 0$ beginning with the starting point $x_0 = 2$. Does something interesting happen?
   (c) Make a sketch of the graph of $f$ and explain what you observed in part b).

4. (a) Let $f(x) = \frac{x^3}{3} + 1$. Calculate the derivative $f'(x)$. Give an anti-derivative of $f'(x)$. Give an anti-derivative that is different from $f(x)$.
   (b) Let $g(x) = x^2 + 1$. Let $G(x)$ be any anti-derivative of $g$. What is $G'(x)$?

5. Find $f$ given that $f'(x) = \sin(x) - \sec(x) \tan(x)$, $f(\pi) = 1$.

6. Find $g$ given that $g''(t) = -9.8$, $g'(0) = 1$, $g(0) = 2$.

   On the surface of the earth, the acceleration of an object due to gravity is approximately $-9.8 \text{ m/s}^2$. What situation could we describe using the function $g$? Be sure to specify what $g$ and its first two derivatives represent.

7. A small rock is dropped from a bridge and the splash is heard 3 seconds later. How high is the bridge?

8. Let $f$ be a function on the domain $(-\infty, \infty)$ that satisfies $(f')^2 = 1$. This is an example of a differential equation. Suppose also that we are given an initial value condition $f(0) = 1$.
   (a) Show that this does not have a unique solution by finding two different functions that satisfy both conditions.
   (b) What does the fact that there are multiple solutions say about this as a model for physical phenomena?

9. Find a function $f(x)$ such that $f'(x) = f(x)$. Find the solution, given initial condition $f(0) = \pi$.

10. Let $f(x) = 1/x$, $F(x) = \ln(|x|)$, and
    $$G(x) = \begin{cases} 
    \ln(x), & x > 0 \\
    \ln(-x) + 8, & x < 0.
    \end{cases}$$
    (a) Is $F$ an anti-derivative of $f$? Is $G$ an anti-derivative of $f$? Is $F - G$ equal to a constant?
    (b) Does Theorem 1 on page 275 imply that $F - G$ is constant? Is the theorem wrong?
1. Use Newton’s Method to estimate $\sqrt[3]{25}$ to four decimal places.

2. Find constants $c_1$ and $c_2$ such that $F(x) = c_1xe^x + c_2e^x$ is an antiderivative of $f(x) = xe^x$.

3. Find an antiderivative for $f(x) = |x|$.

4. Show that $F(x) = \frac{x^{n+1} - 1}{n + 1}$ is an antiderivative of $y = x^n$ for $n \neq -1$. Then use L’Hôpital’s Rule to prove that

$$\lim_{n \rightarrow -1} F(x) = \ln(x)$$

In this limit, $x$ is fixed and $n$ is the variable. This result shows that, although the Power Rule breaks down for $n = -1$, the antiderivative of $y = x^{-1}$ is a limit of antiderivatives of $x^n$ as $n \rightarrow -1$. 