REMKS ON BHASKARA'S APPROXIMATION TO THE SINE OF AN ANGLE

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Bhaskara's rational approximation to the sine of an angle quoted in Mr. Inamdar's paper can be written \( \sin \frac{\pi}{n} = \frac{16(n-1)}{5n^2-4n+4} \). It is interesting to note that this is the best rational approximation that can be devised. Assuming that \( \frac{a+b'n+c'n^2}{a'+b'n+c'n^2} \) reflects the following properties of \( \sin \frac{\pi}{n} \), when \( n \geq 1 \) viz.

(i) \( \sin \frac{\pi}{n} = \sin \frac{\pi}{m} \) where \( m = \frac{n}{n-1} \),

(ii) \( \frac{\pi}{n} \to 0 \) as \( n \to \infty \),

(iii) \( \sin \frac{\pi}{n} = 0, 1, \frac{1}{2} \) when \( n = 1, 2, 6 \) respectively,

we have

(iv) \( \frac{(n-1)(n-a-b)}{a'-(b'+2a')n+n^2(a'+b'+c')} = \frac{a+b'n+c'n^2}{a'+b'n+c'n^2} \); and

(v) \( c=0, \ a+b=0, \ a'+2b'+4c'=a+2b, \ a'+6b'+36c'=2a+12b. \)

Using (v) in (iv) we have \( a'+b'=0 \), so that the last two equations in (v) may be reduced to \( 4c'-a'=b \)

\[ 36c'-5a'=10b \]

and hence \( c' = \frac{5}{16} b, a' = \frac{b}{4} \) and we get the approximation

\[ \frac{n-1}{\left( \frac{5}{16} n^2 - \frac{1}{4} n + \frac{1}{4} \right)} \] or \( \frac{16(n-1)}{5n^2-4n+4} \)

or more elegantly \( \frac{n^2-(n-2)^2}{n^2+\frac{1}{4}(n-2)^2} \),

which is Ganesh's* variant.

Inversely we can express \( n \) in terms of \( \sin \frac{\pi}{n} \) in the form

\[ 1 - \frac{2}{n} = \sqrt{\frac{1 - \sin \frac{\pi}{n}}{1 + \frac{1}{4} \sin \frac{\pi}{n}}} \]

when \( n > 2 \), a formula which enables us to calculate readily the angle with a given sine.

* Ganesh is an Indian Astronomer of the 16th century who wrote much to simplify Astronomy and popularise it.