We list below topics for the second exam. As before, you are permitted to bring a sheet of notes on letter size paper (both sides).

Also note that the exam is at night 5:00-7:00 PM.

1. Review the formulas for the curvature of a space curve as well as the standard vectors $\mathbf{T}, \mathbf{B}, \mathbf{N}$ associated to points of the curve. Learn how the equations of the tangent line to a space curve are found.

   Learn how to use the same calculations to analyze the motion of a particle in space. In particular, learn the tangential and normal components of the acceleration and the osculating plane.

   Learn how to tell if the curve (or the path of the particle) lies in a plane.

2. Learn the meaning of the limit of a function, the idea of continuity and differentiability, both at points and over a region. While I will not ask for complicated calculations, you should review examples to illustrate each of the concepts.

3. Learn how the partial derivatives are computed, for both the explicit and implicit functions. You should also be able to use the chain rule as needed.

   Understand that for any function $f(x, y, z)$, you can write the universal derivative $D(f) = f_x D(x) + f_y D(y) + f_z D(z)$ and that most of the formulas are obtained by taking special meaning for “D”. If you replace D by d, then you get the standard differential of the function.

4. Given any function, there is a linear function associated to it, called the linear approximation (at a point). The approximation is constructed to give a reasonable approximation of the function near a chosen point. This concept is very important in practical applications and be sure to learn it well. Many times, there are word problems which lead to constructing the approximation to a suitable function.

5. The tangent plane to a surface $F(x, y, z) = 0$ has the normal equal to $\nabla F$ evaluated at the desired point. This leads to a simple formula for the tangent plane and indeed, the tangent plane represents the graph of the linear approximation! Learn how to compute and analyze the tangent planes for implicit surfaces $F(x, y, z) = 0$ as well as graphs of a function $z = f(x, y)$.

6. A directional derivative of a function $f$ in a direction $u$ is $\nabla f \cdot u$, provided $u$ is normalized to a unit vector. The directional derivative is largest in the direction of $\nabla f$ evaluated at the desired point and is the smallest in the opposite direction. It is zero along the tangent directions for level surfaces (or curves etc.).

7. Some of the following topics may not appear on the current exam. More details shall be explained later.
• If $\nabla(f)$ is zero at a point, then it is called a critical point. In this case directional derivatives appear to be zero and we need to use second derivatives for finding the behavior of the function.

• The point is a local max/min or a saddle point depending on a suitable second derivative test. This test can also fail, if a certain determinant of second order derivatives evaluates to 0. Suitable generalizations have to be made in higher dimensions.

• This gives us a quick way of finding local and absolute max/min for a continuous function. Unlike one variable, the boundary calculations can be complicated, since the boundaries can be entire curves!

• Lagrange Multipliers are used to calculate max/min of functions restricted to surfaces.