Instructions: This is a written project and you are expected to write down detailed explanations and arguments to justify your calculations. If you wish to use a certain formula, be sure to state it clearly and give a brief justification for it. 

Just a numerical answer, even when correct, will receive little credit.

The problem: Consider the function

\[ f(x, y) = \frac{x^4y}{x^8 - yx^4 + y^2} \]

This is an interesting example where the limit as \((x, y) \to (0, 0)\) does not exist, even though the limit along any line and parabola does.

Answer the following questions.

(1) 3 points.
Show that the limit along any line \(\vec{r}(t) = <at, bt>\) when \(t \to 0\) exists and equals 0.
Answer:

(2) 3 points
Show that the limit along any parabola \(\vec{c}(t) = <at, bt^2>\) when \(t \to 0\) exists and equals 0.
Answer:

(3) 1 point
Calculate \(f(x, y)\) at the points \((10^{-1}, 10^{-4}), (10^{-5}, 10^{-20}), (10^{-20}, 10^{-80})\). Do not use a calculator.
Answer:

(4) 3 points
Show that \(\lim_{(x,y) \to (0,0)} f(x, y)\) does not exist. Hint: Compute the limit along the curve \(y = x^4\).
Answer: