

CHIP FIRING EXERCISES 1

- (1) Let G be a tree. Show that any two adjacent vertices of G are linearly equivalent. Conclude that *any* two vertices of G are linearly equivalent, and the Jacobian of G is trivial.
- (2) Let G be a cycle with n vertices. Label the vertices of G counterclockwise with the elements of $\mathbb{Z}/n\mathbb{Z}$. Show that the map $\text{Div}(G) \rightarrow \mathbb{Z}/n\mathbb{Z}$ given by

$$\sum_{i=0}^{n-1} a_i v_i \mapsto \sum_{i=0}^{n-1} a_i i \pmod{n}$$

is invariant under linear equivalence. Use this to prove that $\text{Jac}(G) \cong \mathbb{Z}/n\mathbb{Z}$.

- (3) Let G_1 and G_2 be graphs, and let G be the graph obtained by connecting a single vertex of G_1 to a single vertex of G_2 by an edge. Show that

$$\text{Jac}(G) \cong \text{Jac}(G_1) \times \text{Jac}(G_2).$$