MA 137
Calculus I with Life Science Applications
FIRST MIDTERM

Fall 2021
09/21/2021

Name: Answer Key

Sect. #: ______________

Do not remove this answer page — you will return the whole exam. No books or notes may be used. Use the backs of the question papers for scratch paper. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The **first part of the exam** consists of 12 multiple choice questions, each worth 5 points. Record your answers on this page by filling in the box corresponding to the correct answer. For example, if (a) is correct, you must write

■a □b □c □d □e

Do not circle answers on this page, but please do circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on **both** this page **and** in the body of the exam.

The **second part of the exam** consists of four open-response questions and one bonus question. When answering these questions, check your answers when possible. Clearly indicate your answer and the reasoning used to arrive at that answer. *Unsupported answers may receive NO credit.*

1. □a □b □c □d ■e

2. □a ■b □c □d □e

3. □a □b ■c □d □e

4. □a □b □c ■d □e

5. □a □b ■c □d □e

6. □a □b □c □d ■e

7. ■a □b □c □d □e

8. □a ■b □c □d □e

9. □a □b □c ■d □e

10. □a □b □c □d ■e

11. □a ■b □c □d □e

12. □a □b □c ■d □e

GOOD LUCK!

QUESTION	SCORE	OUT OF
Multiple Choice		60 pts
13.		10 pts
14.		10 pts
15.		10 pts
16.		10 pts
Bonus.		10 pts
TOTAL		100 pts

Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table:

Sections #	Time/Lecture Location	Lecturer
001-010	MWF 10:00 am - 10:50 am, CB 106	Alberto Corso

Section #	Time/ Recitation Location	TA
001	TR 08:00-08:50 AM, CB 339	Nicholas Arsenault
002	TR 09:00-9:50 AM, CB 339	
003	TR 10:00-10:50 AM, CB 339	Katherine (Kat) Henneberger
004	TR 11:00-11:50 AM, CB 339	
005	TR 12:00-12:50 PM, CB 339	Faith Hensley
006	TR 01:00-01:50 PM, CB 339	
007	TR 12:00-12:50 PM, CB 341	Michael Morrow
008	TR 01:00-01:50 PM, CB 341	
009	TR 02:00-02:50 PM, CB 339	Karen Reed
010	TR 03:00-03:50 PM, CB 339	

1. For an aqueous solution of hydrochloric acid (HCl) the pH was found to be 4.18. What is the concentration of hydrogen ions in this solution?

[Recall that the pH of a liquid is a measure of how acidic or basic it is. It is given by the formula pH $= -\log[H^+]$, where $[H^+]$ is the concentration of hydrogen ions in the liquid and $\log = \log_{10}$]

$4.18 = -\log[H^+]$ thus $-4.18 = \log[H^+]$

and after taking exponential base 10 of both sides we get

$$\underbrace{10^{-4.18}}_{\text{rewrite with a whole integer exponent}} = 10^{\log[H^+]} = [H^+]$$

$$10^{0.82} \cdot 10^{-5} = [H^+]$$

$10^{0.82} \approx 6.6069$

Possibilities:

(a)	1.4×10^{-3}
(b)	2.2×10^{-3}
(c)	3.2×10^{-4}
(d)	4.8×10^{-4}
(e)	$\boxed{6.6 \times 10^{-5}}$

2 Find the equation of the line parallel to the line given by the equation $x - 7y = 15$ and passing through the point $(2, 5)$.

$x - 7y = 15$ can be written as $7y = x - 15$

OR $y = \frac{1}{7}(x - 15)$. Thus the slope is $\frac{1}{7}$

The new line has the same slope and goes through $P(2,5)$

$\therefore \quad y - 5 = \frac{1}{7}(x - 2)$

OR $y - 5 = \frac{x-2}{7}$

Possibilities:

(a)	$y - 5 = x/7$
(b)	$\boxed{y - 5 = (x - 2)/7}$
(c)	$y - 5 = 7(x - 2)$
(d)	$y = 15x/2$
(e)	None of the above

3. Consider the function $f(x) = 2x$. If you evaluate and simplify the expression

$$\frac{f(x+h) - f(x)}{h}$$

you obtain

$$\frac{2(x+h) - 2x}{h}$$

$$= \frac{\cancel{2x} + 2h - \cancel{2x}}{h}$$

$$= \frac{2h}{h}$$

$$= \boxed{2}$$

Possibilities:

(a) $\frac{2(h-x)}{h}$

(b) $2 - x$

$\boxed{\text{(c)} \quad 2}$

(d) 1

(e) 0

4. Which function below is equal to 5^x?

$$5^x = e^{\ln(5^x)}$$

$$= e^{x \ln 5}$$

Possibilities:

(a) e^{5x}

(b) $(\ln(x))^5$

(c) $e^{5\ln(x)}$

$\boxed{\text{(d)} \quad e^{x\ln(5)}}$

(e) $5^{\ln(x)}$

5. Let $f(x) = \dfrac{4x+3}{5x-2}$. Which value of x is **not** in the domain of $f^{-1}(x)$?

(**Hint:** First find a formula for $f^{-1}(x)$.)

$$y = \frac{4x+3}{5x-2}$$ We need to solve for x.

$$y(5x-2) = 4x+3 \iff 5xy - 2y = 4x+3$$
$$\iff 5xy - 4x = 3+2y \iff x(5y-4) = 3+2y$$

$$\therefore \quad x = \frac{3+2y}{5y-4}$$ so that after switching $x \leftrightarrow y$

$$f^{-1}(x) = \frac{3+2x}{5x-4}$$

$x = 4/5$ is not in the domain

Possibilities:

(a) 2/5

(b) $-2/5$

$\boxed{\text{(c) } 4/5}$

(d) $-3/4$

(e) None of the above

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6. Suppose that f is a function of the form $f(x) = A\,e^{kx}$ such that

$$f(1) = 5 \quad \text{and} \quad f(2) = 15.$$

Find A and k.

$$5 = Ae^{k} \qquad 15 = Ae^{2k}$$

$$\boxed{5/e^{k} = A = 15/e^{2k}}$$

This leads to $$\frac{e^{2k}}{e^{k}} = \frac{15}{5}$$ OR $\boxed{e^{k} = 3}$

Thus $\boxed{k = \ln 3}$

$$\therefore \quad A = \frac{5}{e^{k}} = \frac{5}{e^{\ln 3}} = \boxed{\frac{5}{3}}$$

Possibilities:

(a) $A = 1$ and $k = 3$

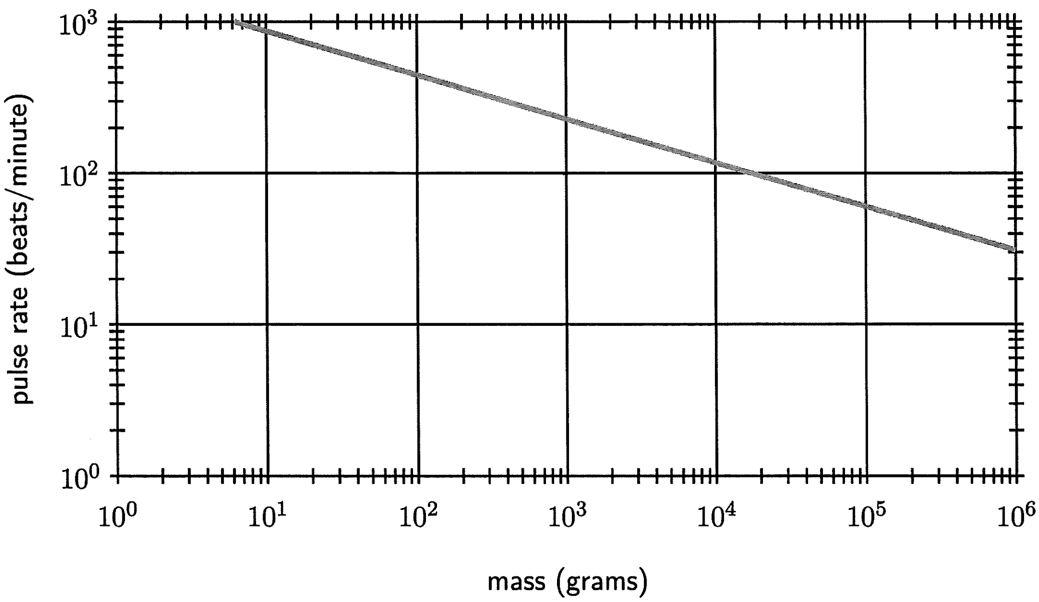
(b) $A = 3$ and $k = 3$

(c) $A = 5$ and $k = 3$

(d) $A = 3/5$ and $k = \ln(3)$

$\boxed{\text{(e) } A = 5/3 \text{ and } k = \ln(3)}$

7. The following log-log plot gives resting heart rates for various animals.



The functional relationship between the mass M of an animal and its pulse rate P best describing the above data is:

Possibilities:

(a) $P = 1,700 \cdot M^{-0.29}$

(b) $P = 1,700 \cdot M^{0.29}$

(c) $P = -0.29M + 1,700$

(d) $P = 1,700 \cdot 10^{-0.29 \cdot M}$

(e) None of the above.

8. Find a formula for the general term a_n of the sequence

$$\frac{-1}{2}, \frac{1}{3}, \frac{-1}{4}, \frac{1}{5}, \frac{-1}{6}, \ldots$$

starting with a_0.

Possibilities:

this has the opposite signs.

(a) $a_n = \dfrac{(-1)^n}{n+2} \quad n \geq 0$

(b) $a_n = \dfrac{(-1)^{n+1}}{n+2} \quad n \geq 0$

(c) $a_n = \dfrac{(-1)^{n+1}}{n+2} \quad n \geq 1$ this starts with a_1

(d) $a_n = \dfrac{1}{n+2} \quad n \geq 0$ ← this has always a positive sign

(e) $a_n = \dfrac{(-1)^n}{n} \quad n \geq 1$ this starts with a_1

For #7

We have a line in a log-log plot. Thus the relationship between quantities M and P is given by a power law:

$$P = A \cdot M^p$$

for some A and p to be determined. Note the p is the slope of the line. Since the slope is negative $\boxed{(a)}$ seems the correct choice.

Let's check. From the graph we know that $(6, 10^3)$ and $(10^6, 30)$ are actual values observed. Thus

$$10^3 = A \cdot 6^p \quad \text{and} \quad 30 = A \cdot (10^6)^p$$

$$\frac{10^3}{6^p} = A = \frac{30}{(10^6)^p} \iff \left(\frac{10^6}{6}\right)^p = \frac{30}{10^3}$$

Take "log" to obtain $p \cdot \log\left(\frac{10^6}{6}\right) = \log\left(\frac{30}{10^3}\right)$

OR $$p = \frac{\log\left(30/10^6\right)}{\log\left(10^6/6\right)} \approx -0.2916 \quad \text{and}$$

$$A = \frac{10^3}{6^{-0.2916}} \approx 1{,}686.19$$

Hence $\boxed{P \approx 1700 M^{-0.2916}}$

9. The sequence $\{a_n\}$ is recursively defined by

$$a_{n+1} = \frac{1}{4}a_n + \frac{3}{4} \qquad a_0 = 2.$$

Find a_n for $n = 1, 2, 3, 4$.

$a_0 = 2$

$a_1 = \frac{1}{4}a_0 + \frac{3}{4} = \frac{2}{4} + \frac{3}{4} = \frac{5}{4}$

$a_2 = \frac{1}{4}a_1 + \frac{3}{4} = \frac{1}{4}\left(\frac{5}{4}\right) + \frac{3}{4} = \frac{17}{16}$

$a_3 = \frac{1}{4}a_2 + \frac{3}{4} = \frac{1}{4}\left(\frac{17}{16}\right) + \frac{3}{4} = \frac{65}{64}$

$a_4 = \frac{1}{4}a_3 + \frac{3}{4} = \frac{1}{4}\left(\frac{65}{64}\right) + \frac{3}{4} = \frac{257}{256}$

Possibilities:

(a) $\frac{3}{4}, \frac{8}{4}, \frac{11}{4}, \frac{14}{4}$

(b) $\frac{5}{4}, \frac{8}{16}, \frac{11}{64}, \frac{14}{256}$

(c) $\frac{5}{4}, \frac{17}{16}, \frac{20}{64}, \frac{257}{256}$

(d) $\boxed{\frac{5}{4}, \frac{17}{16}, \frac{65}{64}, \frac{257}{256}}$

(e) $\frac{3}{4}, \frac{17}{16}, \frac{65}{64}, \frac{257}{256}$

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10. Find all fixed points of the recursive sequence

$$a_{n+1} = \sqrt{2a_n} \qquad a_0 = 1$$

and use a table or other reasoning to decide which fixed point is the limiting value for the given initial condition.

We need to solve the equation $a = \sqrt{2a}$ OR

$a^2 = 2a \iff a^2 - 2a = 0 \quad a(a-2) = 0$

So the fixed points are $\boxed{\hat{a} = 0 \text{ and } \hat{a} = 2}$.

Let's compute a few values $a_0 = 1$

$a_1 = \sqrt{2a_0} = 1.4142$

$a_2 = \sqrt{2a_1} = \sqrt{2\sqrt{2}} = 1.6818$

$a_3 = \sqrt{2a_2} = \sqrt{2\sqrt{2\sqrt{2}}} \simeq 1.8340$

it seems $\boxed{\lim_{n\to\infty} a_n = 2}$

Possibilities:

(a) Two fixed points $\hat{a} = 0, 2$; $\lim_{n\to\infty} a_n$ does not exist

(b) There are no fixed points; $\lim_{n\to\infty} a_n$ does not exist

(c) Two fixed points $\hat{a} = 0, 2$; $\lim_{n\to\infty} a_n = 0$

(d) One fixed point $\hat{a} = 2$; $\lim_{n\to\infty} a_n = 2$

(e) $\boxed{\text{Two fixed points } \hat{a} = 0, 2;\ \lim_{n\to\infty} a_n = 2}$

11. After computing the value of $f(x) = \dfrac{\cos(x) - 1}{x^2}$ for values of x close to 0, you conclude that

$$\lim_{x \to 0} \frac{\cos(x) - 1}{x^2}$$

is equal to

x	$\dfrac{\cos(x)-1}{x^2}$
-0.1	-0.49958
-0.01	-0.4999
-0.001	-0.4999
0.001	-0.4999
0.01	-0.4999
0.1	-0.49958

the calculator must be in radian mode

The table suggests that

$$\boxed{\lim_{x \to 0} \frac{\cos(x)-1}{x^2} = -0.5}$$

Possibilities:

(a) -1
(b) -0.5
(c) 0
(d) 0.5
(e) 1

12. Find the value of the limit

$$\lim_{x \to 3} \frac{x-3}{x^2 - 2x - 3}$$

using the substitution Theorem we get $\frac{0}{0}$. However

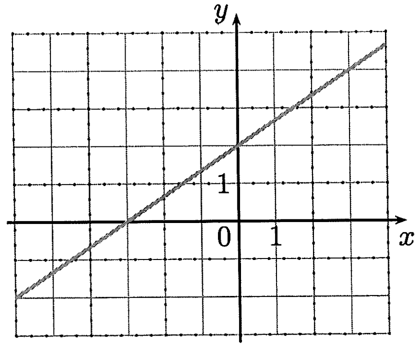
$$\lim_{x \to 3} \frac{\cancel{x-3}}{(\cancel{x-3})(x+1)} =$$

$$\lim_{x \to 3} \frac{1}{x+1} = \boxed{\frac{1}{4}}$$

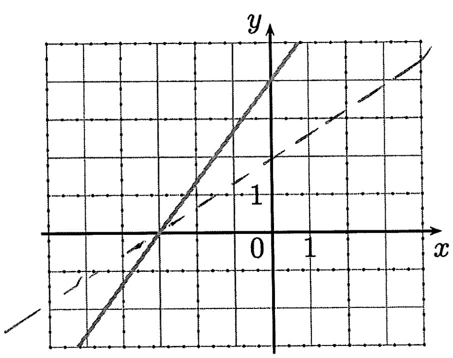
Possibilities:

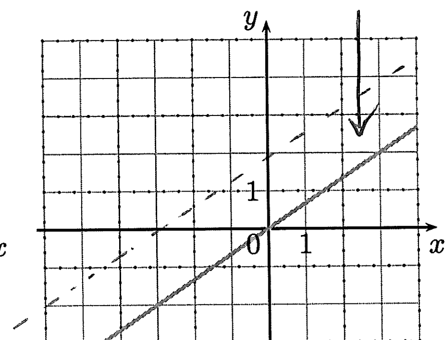
(a) $+\infty$
(b) $-\infty$
(c) The limit does not exists and is not $+\infty$ or $-\infty$
(d) $1/4$
(e) 0

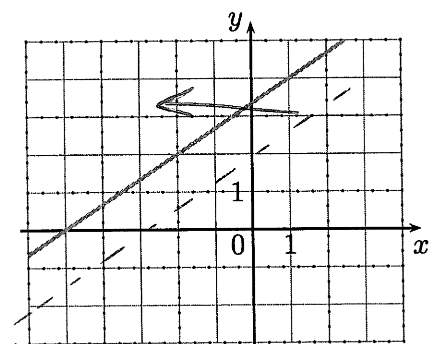
13. If the graph of $f(x)$ is



match the functions $f(x) - 2$, $f(x+2)$ and $2f(x)$ with the graphs below







the y-values have been stretched vertically by a factor of 2

$\boxed{2 \cdot f(x)}$

the graph has been shifted 2 units down

$\boxed{f(x)-2}$

the graph has been shifted 2 units to the left

thus

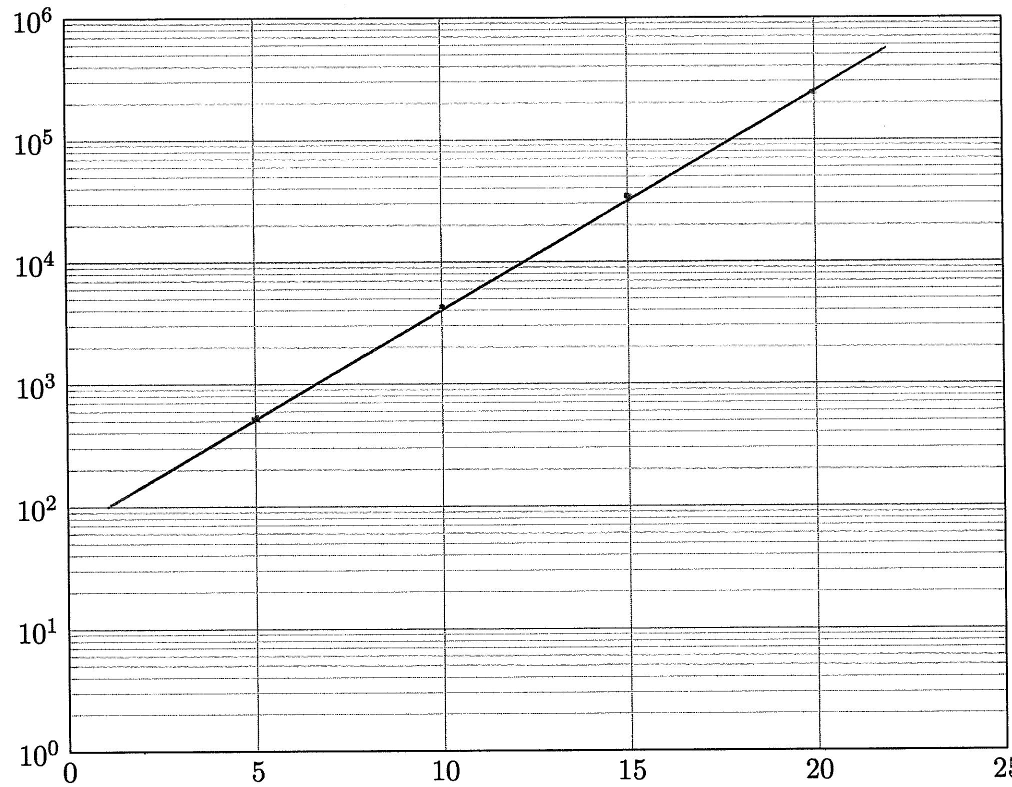
$\boxed{f(x+2)}$

pts: /10

14. When an new species is introduced into an environment there may be no natural predators. In this case, the population may grow very rapidly. Suppose such an invasive species is introduced into a region and the population is measured at several times.

Time in months	5	10	15	20
Population	504	4,032	32,256	258,048

(*a*) Plot this data (as accurately as possible) in the semi log plot below.



(*b*) Find a functional relationship between population and time.

We know that a line in a semi log plot corresponds to a relationship of the form $\boxed{P(t) = B \cdot A^t}$. Since the points are aligned from the graph, let's pick 2 points. Say $(5, 504)$ and $(10, 4032)$ and substitute in the expression to get A and B.

$\boxed{504 = B \cdot A^5}$ and $\boxed{4032 = B \cdot A^{10}}$

pts: /10

We get

$$\frac{504}{A^5} = B = \frac{4032}{A^{10}}$$

So that

$$\frac{A^{10}}{A^5} = \frac{4032}{504}$$

OR $\quad A^5 = 8$

$$\therefore \quad A = 8^{1/5} \approx 1.5157$$

Hence from the first equation we can get B:

$$\frac{504}{\left(8^{1/5}\right)^5} = B = 63$$

$$\therefore \quad \boxed{P(t) = 63 \cdot 8^{t/5}}$$

OR $\quad P(t) = 63\,(1.5157)^t$

You can get essentially the same result by picking other pairs of points.

You can obtain the same result by going through the computation of the line. Using the same points the slope is

$$m = \frac{\log_{10}(4032) - \log_{10}(504)}{10-5}$$

$$= \frac{\log_{10}\left(\frac{4032}{504}\right)}{5} = \frac{1}{5}\log_{10}(8) = \log_{10}(8^{1/5})$$

Hence the equation of the line in the semilog plot is

$$\log_{10} P(t) - \log_{10}(504) = \log_{10}(8^{1/5}) \cdot (t-5)$$

$$\log_{10} P(t) = \left[\log_{10}(8^{1/5})\right] \cdot t - 5\log_{10}(8^{1/5}) + \log_{10}(504)$$

$$= \log_{10}(8^{t/5}) + \log_{10}(8^{-1}) + \log_{10}(504)$$

$$= \log_{10}\left[\frac{504}{8} \cdot 8^{t/5}\right] = \log_{10}(63 \cdot 8^{t/5})$$

$$\therefore \quad P(t) = 63 \cdot 8^{t/5}$$

15. The urination speed of animals increases with the body mass of the mammal. **Yang et al. (2014)** made the following measurements of urination speed (u, measured in ml/s) against animal body mass (M, measured in kg):

Animal	M (kg)	u (ml/s)
Cat	4	3
Dog	20	16
Cow	600	550

It is believed that there is a power-law relationship between urination speed and body mass, namely

$$u = C \cdot M^p$$

for some constants C and p.

(a) Using the given data, plot the corresponding relationship between M and u in the loglog plot below.

(b) Using the given data, estimate the parameters C and p.

(c) Estimate the urination speed u for a human adult (you can assume that $M = 80$ kg).

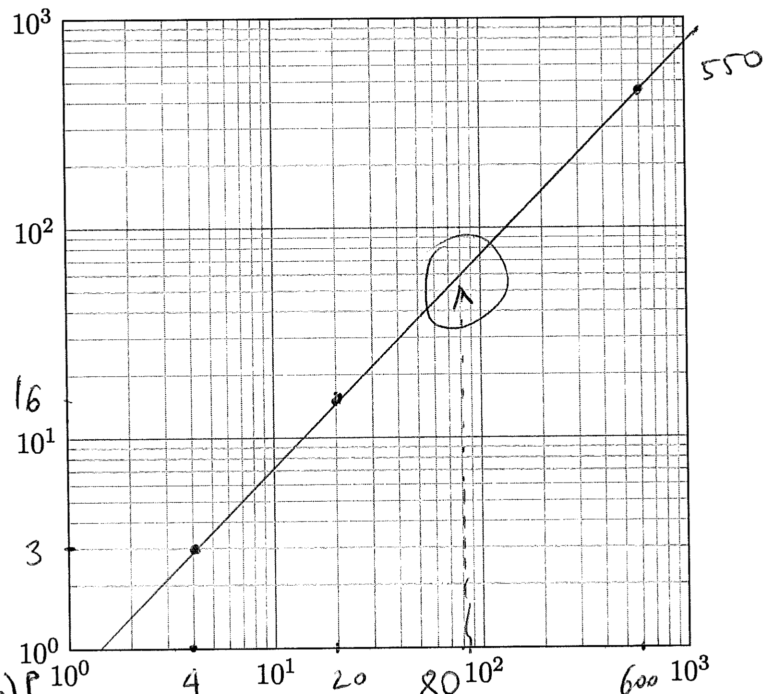
from the picture it seems that the urination speed of a human is between 60 to 70 ml/s.

(b) We have a line in a log-log plot so it corresponds to a relationship (a) of the form

$u = C \cdot M^p$

Let's use (4, 3) and (600, 550) to find C and p

After substituting we get



$3 = C \cdot 4^p$ & $550 = C\,(600)^p$

pts: /10

Thus

$$\frac{3}{4^p} = C = \frac{550}{600^p}$$

OR $$\left(\frac{600}{4}\right)^p = \frac{550}{3}$$

Take ln (OR log) of both sides to get

$$p \ln\left(\frac{600}{4}\right) = \ln\left(\frac{550}{3}\right)$$

so $$p = \frac{\ln(550/3)}{\ln(150)} \approx \underline{1.04}0048953$$

Thus $$C = 3/4^p \cong \frac{3}{4^{1.04}} \approx 0.7095$$

This gives us $\boxed{u = 0.71 M^{1.04}}$

(c) $$u(80) = 0.71\,(80)^{1.04} \approx \underline{67.68}\ \frac{ml}{s}$$

as we predicted

16. (*a*) (5 pts) Find the limit as n tends to ∞ of the following explicit sequence. Justify your answer

$$\lim_{n \longrightarrow \infty} \frac{(1+3n)^2}{2n^2+1}$$

$$= \lim_{n\to\infty} \frac{1+6n+9n^2}{2n^2+1} = \lim_{n\to\infty} \frac{(1+6n+9n^2)\cdot \frac{1}{n^2}}{(2n^2+1)\,\frac{1}{n^2}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{n^2}+\frac{6}{n}+9}{2+\frac{1}{n^2}} \underset{\text{use limit laws}}{=} \frac{\lim_{n\to\infty}\left(\frac{1}{n^2}\right)+\lim_{n\to\infty}\left(\frac{6}{n}\right)+9}{2+\lim_{n\to\infty}\frac{1}{n^2}}$$

$$= \boxed{9/2} = \underline{4.5}$$

(*b*) (5 pts) Write the first five terms of the recursion

$$s_1 = 1 \qquad s_n = 4s_{n-1} + 2 \quad \text{for} \quad n \geq 2.$$

$S_1 = 1$

$S_2 = 4 \cdot S_1 + 2 = 4(1) + 2 = 6$

$S_3 = 4 \cdot S_2 + 2 = 4(6) + 2 = 26$

$S_4 = 4 \cdot S_3 + 2 = 4(26) + 2 = 106$

$S_5 = 4 S_4 + 2 = 4(106) + 2 = 426$

pts: /10

Bonus. (*a*) (5 pts) Compute: $$\lim_{h\to 0}\frac{\sqrt{h+1}-1}{h}$$

direct substitution given $\lim_{h\to 0}\frac{\sqrt{h+1}-1}{h}$

$$= \frac{0}{0} \quad . \quad \lim_{h\to 0}\frac{(\sqrt{h+1}-1)}{h}\cdot\frac{(\sqrt{h+1}+1)}{(\sqrt{h+1}+1)} =$$

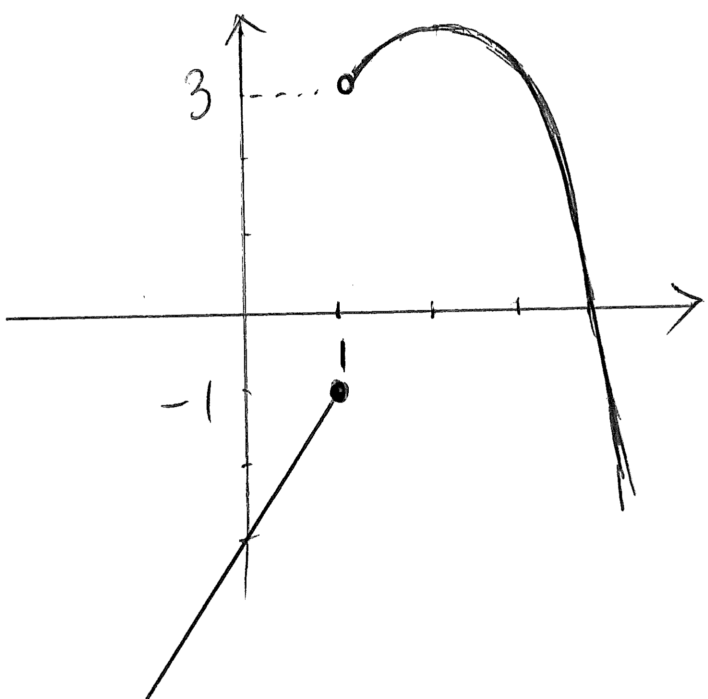
$$= \lim_{h\to 0}\frac{(\sqrt{h+1})^2-1}{h\cdot(\sqrt{h+1}+1)} = \lim_{h\to 0}\frac{\cancel{h}+\cancel{1}-\cancel{1}}{\cancel{h}(\sqrt{h+1}+1)} = \lim_{h\to 0}\frac{1}{\sqrt{h+1}+1}$$

$$= \boxed{1/2}$$

(*b*) (5 pts) Consider the function

$$f(x) = \begin{cases} 2x-3 & x \le 1 \\ 4x - x^2 & x > 1 \end{cases}$$

After having drawn the graph of $f(x)$, compute: $f(1)$, $\lim_{x\to 1^-} f(x)$, $\lim_{x\to 1^+} f(x)$, $\lim_{x\to 1} f(x)$.



$* \; f(1) = 2(1) - 3 = \boxed{-1}$

$* \; \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(2x-3)$

$= 2\left(\lim_{x\to 1^-} x\right) - 3$

$= 2\cdot(1) - 3 = \boxed{-1}$

$* \; \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(4x - x^2)$

$= 4\lim_{x\to 1^+} x - \left(\lim_{x\to 1^+} x\right)^2$

$= 4(1) - 1^2 =$

Thus

$\lim_{x\to 1} f(x) = \text{D.N.E.}$

pts: $\boxed{3}$ /10