MA 137
Calculus I with Life Science Applications
FIRST MIDTERM

Fall 2020
09/15/2020

Name: _____

Sect. #: _____

Length of exam: Unless you have a DRC accommodation letter, you will have until 7:30 PM on September 15, 2020 to upload a PDF with your answers for the exam in the same group assignment on Canvas where you downloaded the exam. The exam is written so that it should take you at most 2 hours for the exam, allowing 30 minutes to scan and upload the exam as a single PDF on Canvas. Budget your time appropriately as NO extensions will be given.

Students with a DRC accommodation letter will take the exam with the students in sections 001 and 002. Thus they should use the Zoom number for sections 001 and 002. Their exam will end at 8:30 PM if they are allowed 50% extra time or at 9:30 PM if they are allowed 100% extra time.

Submitting your exam: You can annotate the PDF on an e-device, for example on a university issued iPad. Alternatively, you could print the test and write all your solutions on the printed exam. If it is too time consuming and/or impossible to print the test, just write on blank sheets of paper your work for the multiple choice questions and for the open response questions.

Please make sure to write your name and list the correct section number on the front page of your exam. In case you have forgotten your section number, consult the table below.

Please make sure to write your answers for the multiple choice questions either on the second page of the exam or on a single sheet of paper. You should include any supporting work that you deem appropriate for the multiple choice questions. The answers must be in the same order as the multiple choice questions (namely, question 1/answer, question 2/answer, etc,). Similarly, please write your answers to the open response questions on either the exam pages or on separate sheets of paper, making sure your answer pages are scanned in sequential order (answer to problem 13, then answer to problem 14, etc.). *You will be penalized 10 points if you provide the answers in a scrambled order.*

Questions during exam: You will be proctored for the entire exam time by your TA at the following Zoom link from 5 pm to 7:30 pm. You are required to have your camera on during the entire exam. If you need any clarification during the exam please ask a private question in the Zoom chat.

Section	Time/Recitation Location	TA	Zoom number
001	TR 08:00-08:50 AM, CB 240	J. Garagnani	https://uky.zoom.us/j/95901154457
002	TR 09:00-9:50 AM, CB 240		
003	TR 10:00-10:50 AM, CB 242	J. Britt	https://uky.zoom.us/j/92891213047
004	TR 11:00-11:50 AM, CB 242		
005	TR 12:00-12:50 PM, CB 246	W. Rizer	https://uky.zoom.us/j/93064919193
006	TR 01:00-01:50 PM, CB 246		
007	TR 12:00-12:50 PM, CB 244	R. Righi	https://uky.zoom.us/j/91644476535
008	TR 01:00-01:50 PM, CB 244		
009	TR 02:00-02:50 PM, CB 246	M. McCarver	https://uky.zoom.us/j/96785994735
010	TR 03:00-03:50 PM, CB 246		

Restrictions on books, notes, calculators and cell phones: You will return the whole exam with your answers or the sheets that you want us to grade. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. No books or notes may be used. Absolutely no cell phone use during the exam is allowed, except for scanning your exam pages. Make sure to work in a quiet environment.

The **first part of the exam** consists of 12 multiple choice questions, each worth 5 points. Record your answers on this page by filling in the box corresponding to the correct answer. For example, if (a) is correct, you must write

[a] [b] [c] [d] [e]

It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on **both** this page **and** in the body of the exam.

The **second part of the exam** consists of four open-response questions and one bonus question. When answering these questions, check your answers when possible. Clearly indicate your answer and the reasoning used to arrive at that answer. *Unsupported answers may receive NO credit*.

Cheating (Senate Rule 6.3.2): Cheating is a serious offense and will not be tolerated. It will be thoroughly investigated, and might lead to failure in the course or even to expulsion from the university. Cheating is defined by its general usage. It includes, but is not limited to, wrongfully giving, taking, or presenting any information or material by a student with the intent of aiding themselves or another on any academic work which is considered in any way in the determination of the final grade. The fact that a student could not have benefited from an action is not by itself proof that the action does not constitute cheating. Any question of definition shall be referred to the University Appeals Board.

1. [a] [b] [c] [d] [e]

2. [a] [b] [c] [d] [e]

3. [a] [b] [c] [d] [e]

4. [a] [b] [c] [d] [e]

5. [a] [b] [c] [d] [e]

6. [a] [b] [c] [d] [e]

7. [a] [b] [c] [d] [e]

8. [a] [b] [c] [d] [e]

9. [a] [b] [c] [d] [e]

10. [a] [b] [c] [d] [e]

11. [a] [b] [c] [d] [e]

12. [a] [b] [c] [d] [e]

GOOD LUCK!

QUESTION	SCORE	OUT OF
Multiple Choice		60 pts
13.		10 pts
14.		10 pts
15.		10 pts
16.		10 pts
Bonus.		10 pts
TOTAL		100 pts

1. The pH of a liquid is a measure of how acidic or basic it is. The concentration of hydrogen ions in a liquid is labeled $[H^+]$. Use the formula $\text{pH} = -\log[H^+]$ to find the pH level, to the nearest tenth, of a liquid with $[H^+]$ about 3.2×10^{-2}.

Possibilities:

(a) $\text{pH} = -2.5$

(b) $\text{pH} = 2.5$

(c) $\text{pH} = -1.5$

(d) $\text{pH} = 1.5$

(e) $\text{pH} = 3.5$

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2. Find the center $C(h, k)$ and the radius r of the circle defined by the equation

$$2x^2 + 2y^2 + 6x - 8y + 12 = 0.$$

(a) $C(3/2, -2)$ and $r = 1/2$

(b) $C(-3/2, 2)$ and $r = 1/4$

(c) $C(3/2, -2)$ and $r = 1/4$

(d) $C(-3, 4)$ and $r = \sqrt{6}$

(e) $C(-3/2, 2)$ and $r = 1/2$

3. Write the logarithmic expression $2\log_b q + 3\log_b y$ as a single expression:

Possibilities:

(a) $(2+3)\log_b(q+y)$

(b) $\log_b(q^2y^3)$

(c) $\log_b(q^2+y^3)$

(d) $\log_b\left(qy^{2+3}\right)$

(e) $\log_b\left(\dfrac{q^2}{y^3}\right)$

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4. Let $f(x) = \dfrac{5x-2}{4x+3}$. Then the inverse $f^{-1}(x)$ of f is:

Possibilities:

(a) $f^{-1}(x) = \dfrac{4x+3}{5x-2}$

(b) $f^{-1}(x) = \dfrac{2-5x}{3-4x}$

(c) $f^{-1}(x) = \dfrac{5/x-2}{4/x+3}$

(d) $f^{-1}(x) = \dfrac{5-4x}{3x+2}$

(e) $f^{-1}(x) = \dfrac{3x+2}{5-4x}$

5. Consider the functions $f(x) = x + 3$ and $g(x) = \dfrac{1}{x-3}$

What is the simplified form of the composition function $(g \circ f)(x)$?

(a) $g(f(x)) = \dfrac{x+3}{x-3},\ x \neq 3$

(b) $g(f(x)) = \dfrac{1}{x},\ x \neq 0$

(c) $g(f(x)) = \dfrac{3x-8}{x+3},\ x \neq -3$

(d) $g(f(x)) = \dfrac{3x-8}{x-3},\ x \neq 3$

(e) $g(f(x)) = \dfrac{-8}{x},\ x \neq 0$

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6. Let $B(t)$ denote the number of bacteria at time t (measured in hours) in a certain culture. The population grows according to the exponential growth model. Thus

$$B(t) = B_0 e^{rt}$$

for some positive constants B_0 and r.

Suppose the population has a doubling time of 20 hours. Find the constant r.

Possibilities:

(a) $r = \ln 2/20$

(b) $r = 20/\ln 2$

(c) $r = 1/2$

(d) $r = 2$

(e) None of the above

7. The half-life of a certain radioactive material is 85 days. An initial amount of the material has a mass of 801 kg. Write an exponential function that models the decay of this material y at time t. Find how much radioactive material remains after 10 days. Round your answer to the nearest thousandth.

Possibilities:

(a) $y = \frac{1}{2}\left(\frac{1}{801}\right)^{\frac{1}{85}t}$; 0.228 kg

(b) $y = 801\left(\frac{1}{2}\right)^{\frac{1}{85}t}$; 738.273 kg

(c) $y = 801\left(\frac{1}{2}\right)^{85t}$; 0 kg

(d) $y = 2\left(\frac{1}{801}\right)^{\frac{1}{85}t}$; 0.911 kg

(e) $y = 85\left(\frac{1}{2}\right)^{\frac{1}{801}t}$; 84.268 kg

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8. The sequence $\{b_n\}$ is recursively defined by

$$b_{n+1} = 3b_n - 2 \qquad b_0 = 2.$$

Find b_n for $n = 1, 2, 3, 4, 5$.

Possibilities:

(a) $2, 4, 10, 28, 82$

(b) $-2, -8, -26, -80, -242$

(c) $4, 10, 28, 82, 244$

(d) $10, 28, 82, 244, 730$

(e) None of the above

9. Consider the sequence $a_{n+1} = \frac{1}{2}\left(a_n + \frac{9}{a_n}\right)$ with $a_0 = 1.$

Then $\lim_{n\to\infty} a_n$ is

Possibilities:

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

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10. Consider the recursive sequence $a_{n+1} = \frac{9}{a_n} \qquad a_0 = 1.$

Find the fixed points of this recursion and investigate the limiting behavior of a_n.

Possibilities:

(a) Fixed points $\widehat{a} = 1, 9$; $\lim_{n\to\infty} a_n = 9$

(b) Fixed points $\widehat{a} = \pm 3$; $\lim_{n\to\infty} a_n = 3$

(c) Fixed points $\widehat{a} = 1, 9$; $\lim_{n\to\infty} a_n$ does not exist

(d) Fixed points $\widehat{a} = \pm 3$; $\lim_{n\to\infty} a_n$ does not exist

(e) There are no fixed points; $\lim_{n\to\infty} a_n$ does not exist

11. After computing the value of $f(x) = \dfrac{\ln(1-x)+x}{x^2}$ for values of x close to 0, you conclude that

$$\lim_{x\to 0} \frac{\ln(1-x)+x}{x^2}$$

is equal to:

x	$\dfrac{\ln(1-x)+x}{x^2}$
-0.1	
-0.01	
-0.001	
0.001	
0.01	
0.1	

Possibilities:

(a) -1
(b) -0.5
(c) 0
(d) 0.5
(e) 1

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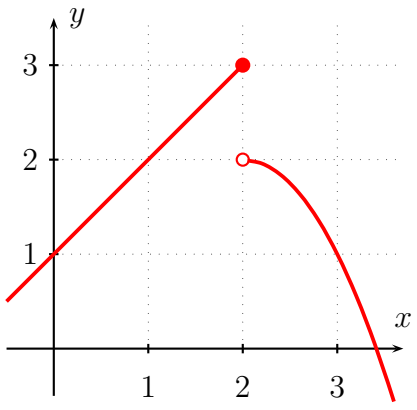
12. Use the graph of f below to determine which of the following statements is true.



Possibilities:

(a) f is continuous at $x = 2$
(b) $\lim_{x\to 2} f(x) = 3$
(c) $\lim_{x\to 2^-} f(x) = 3$
(d) $\lim_{x\to 2^+} f(x) = f(2)$
(e) $\lim_{x\to 2^+} f(x) = 3$

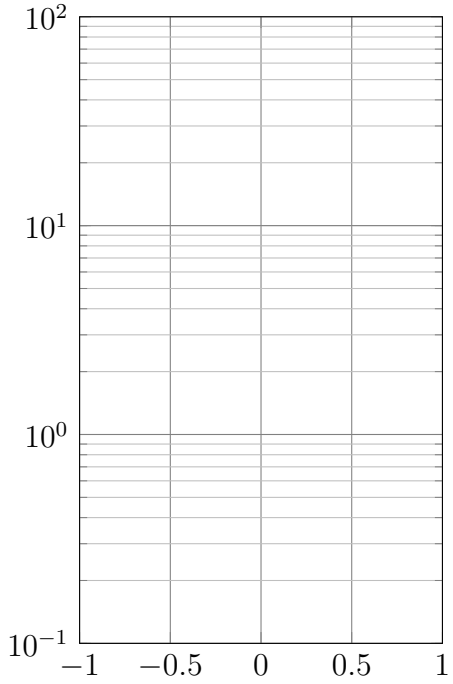
13. The table below is based on a functional relationship between x and y that is either an exponential or a power function:

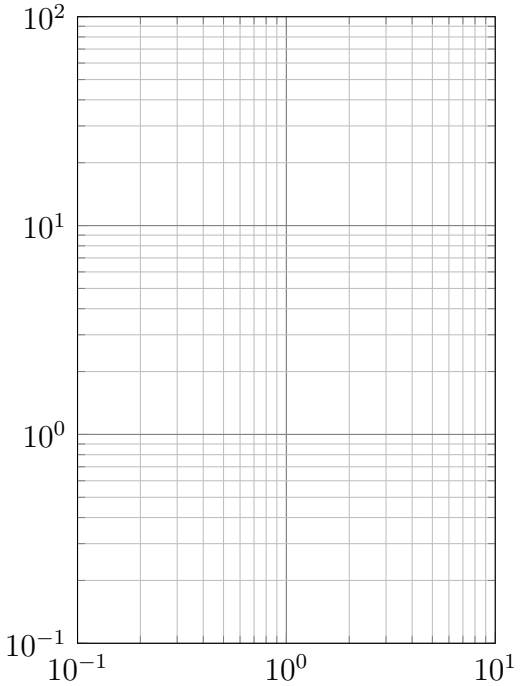
x	y
-1	39.8
-0.5	12.585
0	3.98
0.5	1.2585
1	0.398

Use an appropriate logarithmic transformation and a graph to decide whether the table comes from a power function or an exponential function,

☐ power function ☐ exponential function

and find the functional relationship between x and y: $y =$ _____ .





pts: /10

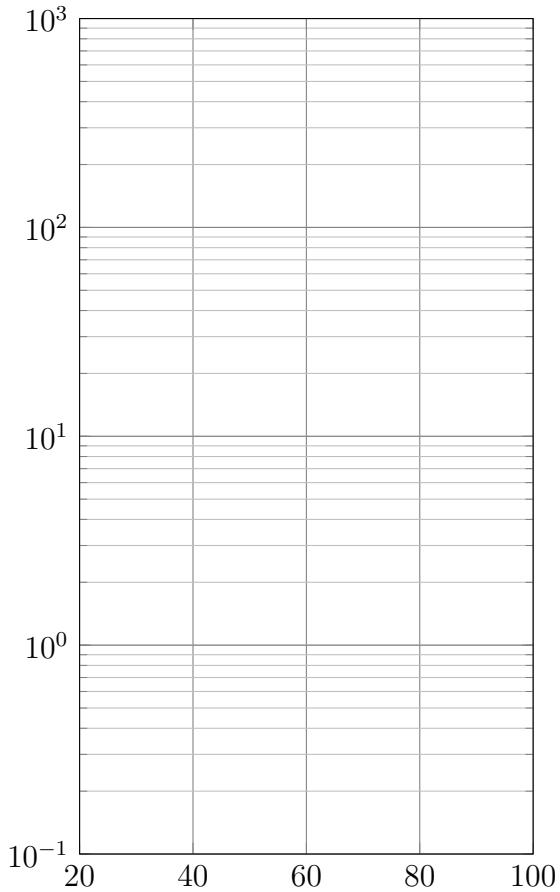
14. (*Adapted from Benton and Harper, 1997*) In vertebrates, embryos and juveniles have large heads relative to their overall body size. As the animal grows older, proportions change; for instance, the ratio of skull length to body length diminishes. That this is the case not only for living vertebrates, but also for fossil vertebrates, is shown by the following example.

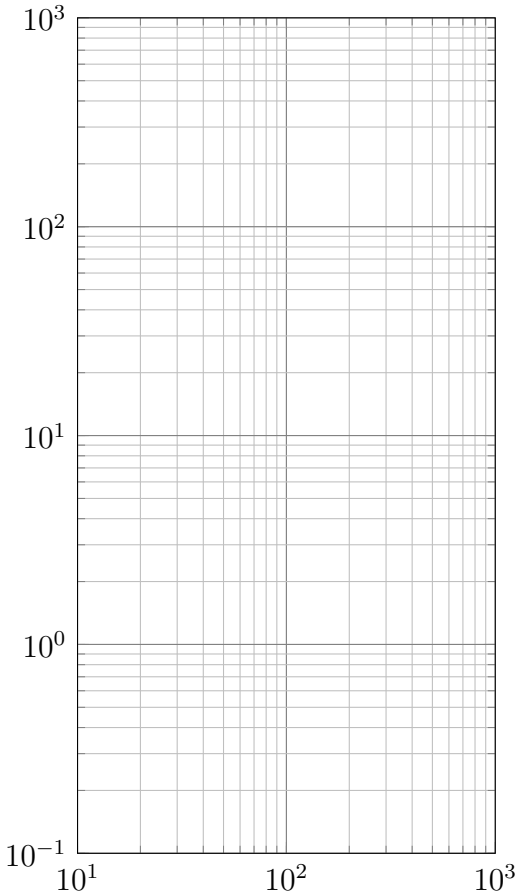
Ichthyosaurs are a group of marine reptiles that appeared in the early Triassic and died out well before the end of the Cretaceous. They were fish shaped and comparable in size to dolphins. In a study of 20 fossil skeletons, the following allometric relationship between backbone length B (measured in cm) and skull length S (measured in cm) was found

$$S = 1.162B^{0.93}$$

(a) Use a suitable logarithmic transformation of B and S so that the resulting relationship is linear. What is the linear relationships that you found?

(b) Graph the resulting linear relationship in the appropriate plot below. Mark your choice clearly and make your plot accurate!





pts: /10

15. (a) Consider a population model with emigration given by

$$p_{n+1} = 1.1p_n - 3,000 \qquad n = 0, 1, 2, \ldots$$

with an initial population of $p_0 = 10,000$.
Find the population at the next three time intervals. Namely, p_1, p_2 and p_3.

(b) Find the limit as n tends to ∞ of the following explicit sequence. Justify your answer:

$$\lim_{n\to\infty} \frac{2n^2 + 2^{-n}}{(5 - 3n)n}$$

pts: /10

16. Compute the following limits. Make sure to justify your answers.

(a) $\lim_{x \to 0} \dfrac{2e^x + \cos x}{2x + 4}$

(b) $\lim_{x \to 3} \dfrac{(3 - x)^2}{9 - x^2}$

pts: /10

Bonus. For what value of the constant c is the function f defined below

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$$

continuous on $(-\infty, \infty)$?

pts: /10