

Do not remove this answer page — you will return the whole exam. No books or notes may be used. Use the backs of the question papers for scratch paper. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The **first part of the exam** consists of 12 multiple choice questions, each worth 5 points. Record your answers on this page by filling in the box corresponding to the correct answer. For example, if (a) is correct, you must write

a    b    c    d    e

Do not circle answers on this page, but please do circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on **both** this page **and** in the body of the exam.

The **second part of the exam** consists of four open-response questions and one bonus question. When answering these questions, check your answers when possible. Clearly indicate your answer and the reasoning used to arrive at that answer. *Unsupported answers may receive NO credit.*

1.  a    b    c    d    e

2.  a    b    c    d    e

3.  a    b    c    d    e

4.  a    b    c    d    e

5.  a    b    c    d    e

6.  a    b    c    d    e

7.  a    b    c    d    e

8.  a    b    c    d    e

9.  a    b    c    d    e

10.  a    b    c    d    e

11.  a    b    c    d    e

12.  a    b    c    d    e

**GOOD LUCK!**

QUESTION	SCORE	OUT OF
<b>Multiple Choice</b>		60 pts
<b>13.</b>		10 pts
<b>14.</b>		10 pts
<b>15.</b>		10 pts
<b>16.</b>		10 pts
<b>Bonus.</b>		10 pts
<b>TOTAL</b>		100 pts

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Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table:

<b>Sections #</b>	<b>Time/Lecture Location</b>	<b>Lecturer</b>
<b>001-010</b>	MWF 10:00 am - 10:50 am, CB 106	Alberto Corso
<b>Section #</b>	<b>Time/ Recitation Location</b>	<b>TA</b>
<b>001</b>	TR 08:00-08:50 AM, CB 339	Nicholas Arsenault
<b>002</b>	TR 09:00-9:50 AM, CB 339	
<b>003</b>	TR 10:00-10:50 AM, CB 339	Katherine (Kat) Henneberger
<b>004</b>	TR 11:00-11:50 AM, CB 339	
<b>005</b>	TR 12:00-12:50 PM, CB 339	Faith Hensley
<b>006</b>	TR 01:00-01:50 PM, CB 339	
<b>007</b>	TR 12:00-12:50 PM, CB 341	Michael Morrow
<b>008</b>	TR 01:00-01:50 PM, CB 341	
<b>009</b>	TR 02:00-02:50 PM, CB 339	Karen Reed
<b>010</b>	TR 03:00-03:50 PM, CB 339	

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1. Let  $B(t)$  denote the number of bacteria at time  $t$  (measured in hours) in a certain culture. The population grows exponential, thus

$$B(t) = B_0 e^{rt}$$

for some positive constants  $B_0$  and  $r$ .

Suppose the population has a doubling time of 3 hours. Find the rate of growth  $r$ .

**Possibilities:**

- (a)  $r = (\ln 2)/3$
- (b)  $r = 3/\ln 2$
- (c)  $r = 1/2$
- (d)  $r = 1/3$
- (e)  $r = \ln(2/3)$

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2. Let

$$f(x) = \begin{cases} -2x + c & x \leq 1 \\ 3x^2 + 2x + 3 & x > 1 \end{cases}$$

For what value(s) of  $c$  is this function continuous?

**Possibilities:**

- (a)  $c = 8$
  - (b)  $c = 10$
  - (c)  $c = 12$
  - (d)  $c = 14$
  - (e)  $c = 16$
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3. Find a function  $f$  and a number  $a$  so that the following limit represents a derivative  $f'(a)$

$$\lim_{h \rightarrow 0} \frac{\ln(e+h) - 1}{h}$$

**Possibilities:**

- (a)  $f(x) = \ln(x)$   $a = 1$
- (b)  $f(x) = \ln(x)$   $a = e$
- (c)  $f(x) = \ln(x+e)$   $a = 1$
- (d)  $f(x) = \ln(x+1)$   $a = e$
- (e) None of the above.

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4. Find the linearization of  $f(x) = \frac{x}{1+x^2}$  at  $x = 2$ .

**Possibilities:**

- (a)  $L(x) = \frac{2}{5}$
  - (b)  $L(x) = \frac{3}{25}x - \frac{1}{25}$
  - (c)  $L(x) = -\frac{3}{25}x + \frac{16}{25}$
  - (d)  $L(x) = x + 2$
  - (e)  $L(x) = \frac{2}{5}x$
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- 
5. In the  $xy$ -plane, the line  $x + y = k$ , where  $k$  is a constant, is tangent to the graph of  $y = x^2 + 3x$ . What is the value of  $k$ ?

**Possibilities:**

- (a) 4
- (b) 2
- (c) 0
- (d)  $-2$
- (e)  $-4$

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6. Suppose a function  $y = y(x)$  is implicitly defined by the equation

$$x^2 + xy + y^2 = 1.$$

Find a formula for  $dy/dx$  in terms of  $x$  and  $y$ .

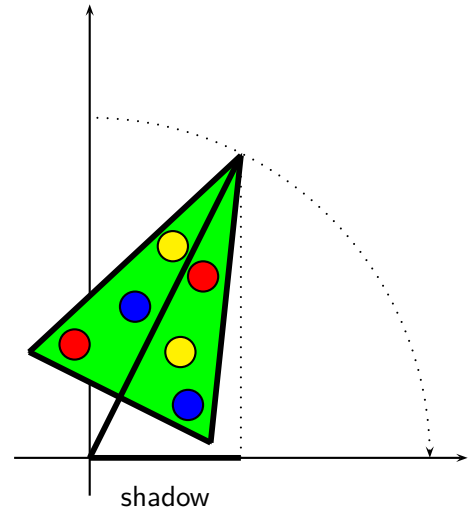
**Possibilities:**

- (a)  $\frac{dy}{dx} = \frac{-(2x + y)}{x + y}$
  - (b)  $\frac{dy}{dx} = \frac{-2x}{x + 2y}$
  - (c)  $\frac{dy}{dx} = \frac{2x + y}{x + 2y}$
  - (d)  $\frac{dy}{dx} = \frac{-(2x + y)}{x + 2y}$
  - (e) None of the above
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7. On Christmas Eve the full moon is directly overhead the town of Whoville, and its 50-foot-tall outdoor Christmas tree. The Grinch wants to ruin the Whos' Christmas so he decides to cut down this Christmas tree. We are watching the dramatic scene of this majestic tree falling over. Since we are mathematically inclined, we observe that at the moment when the top of the tree is 30 feet from the ground it is falling at the rate of 4 feet/sec. At that moment, how rapidly is the shadow of the tree, cast by the moon, lengthening?

**Possibilities:**

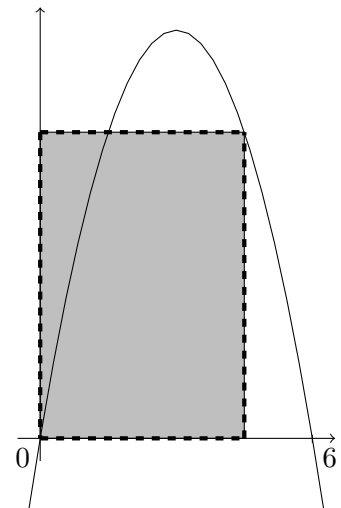
- (a) 1 feet per second
- (b) 2 feet per second
- (c) 3 feet per second
- (d) 4 feet per second
- (e) 5 feet per second



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8. Consider a rectangle that lies in the first quadrant with one vertex at the origin and two of the sides along the coordinates axis. The fourth vertex lies on the parabola  $y = 6x - x^2$ . Find the area of the largest such rectangle.

**Possibilities:**

- (a) 40
- (b) 32
- (c) 27
- (d) 3
- (e) 0



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9. Use a geometric argument to evaluate

$$\int_0^2 \sqrt{4 - x^2} dx$$

**Possibilities:**

- (a)  $4\pi$
- (b)  $3\pi$
- (c)  $2\pi$
- (d)  $\pi$
- (e)  $0$

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10. Let  $f(x) = 2x^2 - x^3$ . Find the largest open interval on which  $f$  is increasing.

**Possibilities:**

- (a)  $(-2, 0)$
  - (b)  $(0, 2)$
  - (c)  $(-\infty, 2/3)$
  - (d)  $(2/3, \infty)$
  - (e)  $(0, 4/3)$
-

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11. Suppose  $G(x) = \int_1^{x^2} t^2 e^t dt$ . Find  $G'(2)$ .

**Possibilities:**

- (a)  $16e^2$
- (b)  $64e^2$
- (c)  $16e^4$
- (d)  $64e^4$
- (e)  $96e^8$

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12. Suppose  $h$  is a continuous function such that

$$h(1) = -2 \quad h'(1) = 2 \quad h''(1) = 2$$

$$h(2) = 6 \quad h'(2) = 5 \quad h''(2) = 13$$

and  $h''$  is continuous everywhere.

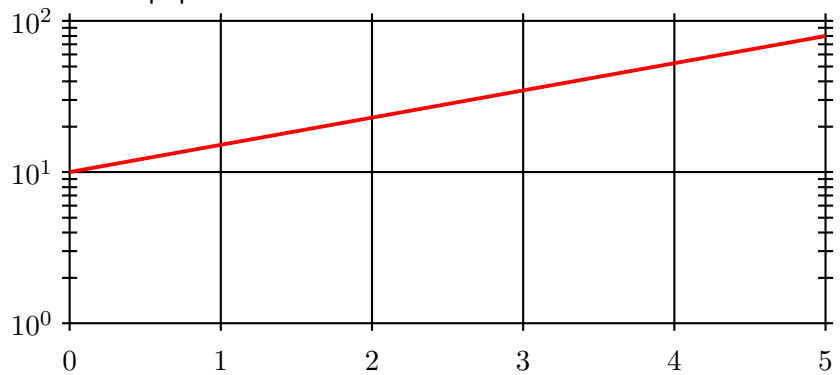
Evaluate:  $\int_1^2 h''(t) dt$ .

**Possibilities:**

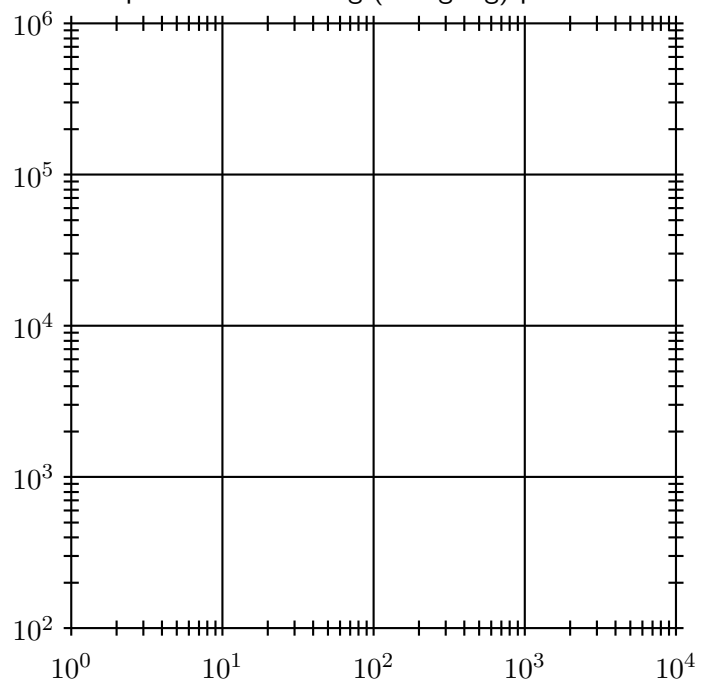
- (a) 3
  - (b) 4
  - (c) 7
  - (d) 8
  - (e) 11
-



13. (a) The graph below is a semilog plot for the size of a population of fruit flies as a function of time. What function describes the size of the population as a function of time?



- (b) The number of calories an active adult dog requires per day is determined by the mass of the dog. The function that describes this relationship is  $C = 125 \cdot M^{3/4}$ , where  $C$  is the number of kilocalories per day and  $M$  is the weight of the dog in kilograms (source: [www.dogfoodadvisor.com](http://www.dogfoodadvisor.com)). Plot this relationship on the double-log (or log-log) plot below<sup>1</sup>.



pts: /10

<sup>1</sup>The plot is meaningful, say, for the values  $1 \leq M \leq 100$ . For ease of plotting, we consider however the larger log-log plot above.

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14. Use l'Hospital's rule to evaluate the following limits:

(a)  $\lim_{x \rightarrow 2} \frac{x^2 - 8x + 12}{4 - x^2}$

(b)  $\lim_{x \rightarrow 0} \frac{\int_0^x e^h dh}{x^2}$

pts: /10

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15. (a) Compute the indefinite integral  $\int \frac{2x^2 - x}{\sqrt{x}} dx$

(b) Solve the initial value problem

$$\frac{dW}{dt} = e^{-3t}$$

for  $t \geq 0$  and  $W(0) = 2$ .

pts: / 10

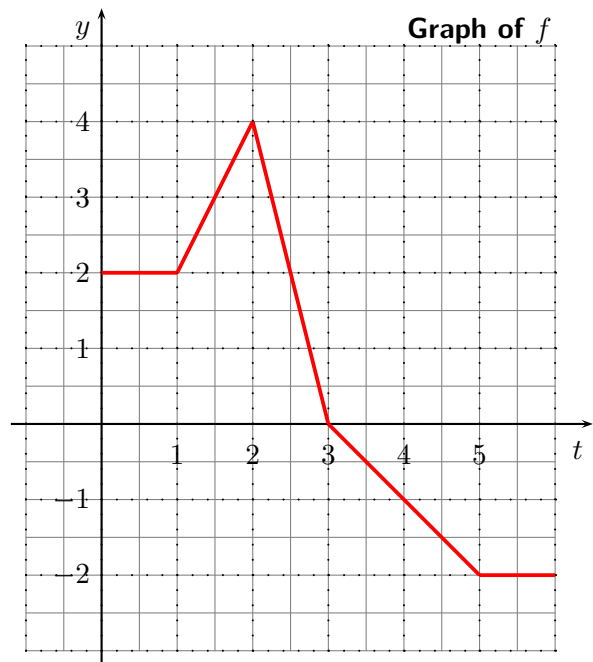
16. Let  $g(x) = \int_0^x f(t) dt$ , where  $f$  is the function whose graph is shown below.

(i) Determine the following values of this new function  $g$ :

$g(0) = \underline{\hspace{2cm}}$        $g(1) = \underline{\hspace{2cm}}$

$g(2) = \underline{\hspace{2cm}}$        $g(3) = \underline{\hspace{2cm}}$

$g(4) = \underline{\hspace{2cm}}$        $g(5) = \underline{\hspace{2cm}}$



(ii) Find the derivative of the function  $g$  and determine the  $x$  value at which  $g$  attains its maximum value.

**Bonus.** (a) Find both fixed points for the recursive sequence:  $a_{n+1} = \frac{a_n^2 + 8}{3a_n}$ .

(b) What does the Stability Criterion say about the fixed points found in (a)?

It is given that  $f'(x) = \frac{x^2 - 8}{3x^2}$  is the derivative of  $f(x) = \frac{x^2 + 8}{3x}$ .

(c) Given the initial value  $a_0 = 1.3$ , compute  $a_1, a_2, a_3, a_4$ , and  $a_5$ :

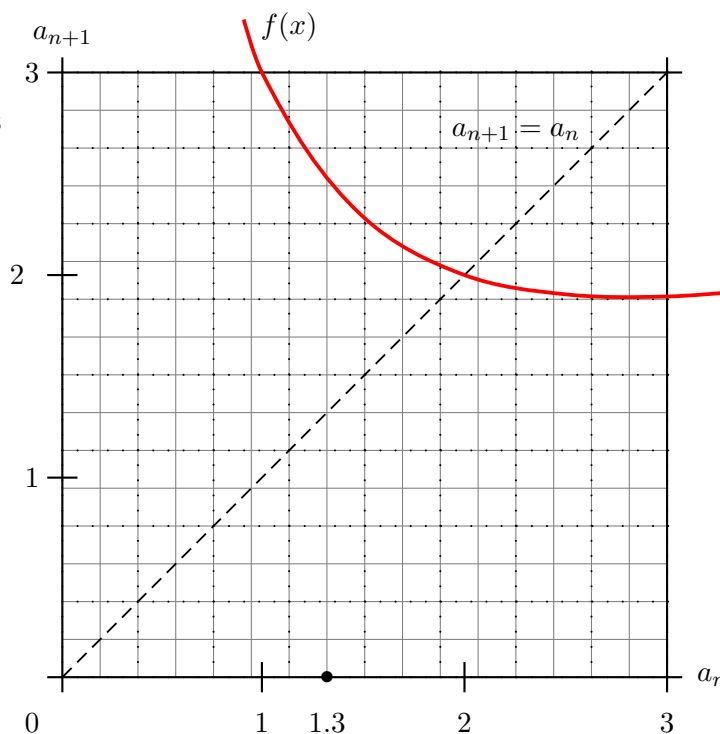
$a_1 =$  \_\_\_\_\_       $a_2 =$  \_\_\_\_\_       $a_3 =$  \_\_\_\_\_

$a_4 =$  \_\_\_\_\_       $a_5 =$  \_\_\_\_\_

(d) Sketch a cobweb graph starting at  $a_0 = 1.3$  on the given plot.

Make sure to mark at least the values  $a_1, a_2, a_3, a_4$ , and  $a_5$  found in (c).

Use it to determine  $\lim_{n \rightarrow \infty} a_n$ .



pts: / 10