The Monty Hall Problem

Lesson Plan

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Goal: The goal of this lesson is to utilize both experimental and theoretical probabilities to determine the best strategy to follow when playing the Monty Hall Game.

Grade and Course: 8th grade, pre-algebra

KY Standards: MA-08-4.4.2 (Data Analysis and Probability: Probability)

Objectives: This activity is intended to be a fun way to apply both experimental and theoretical probability. The Monty Hall Problem is this: there are three doors – behind one door is a new car and behind the other two are goats. The contestant chooses one door and then the host opens one of the *other* two revealing a goat. The contestant may then choose to take the prize behind the door they picked initially or they may switch and take the prize behind other remaining closed door. The question is: to maximize their chances of winning, should they switch doors or stay with their initial pick or does it make no difference?

The best strategy is to switch doors. The theoretical probability of winning is 2/3 in this case versus a 1/3 chance of winning if you stay with your initial pick. This solution is not obvious, however. Many students will reason that after the host opens one door with a goat, since there are two doors remaining – one a winner, one a loser – the contestant has a 50/50 chance of winning whether they switch doors or not. Indeed, some PhDs in math rebutted Marilyn vos Savant's correct solution to the problem with this very argument. The error in this reasoning lies in the fact that the events of this experiment (the contestant's first choice, the door the host opens, the contestant's second choice) are dependent events, so the probabilities of the grand prize being behind either remaining door are not equal. If it hasn't already been introduced, this would be a good opportunity to demonstrate the difference between dependent and independent events to the class.

Resources/materials needed: We used Styrofoam cups to represent the three doors/prize rooms and colored blocks to represent the prizes. You could buy some cheap plastic cars and animals from a dollar store to add pizzazz to the simulation.

Description of Plan: Explain the Monty Hall Problem (described at the beginning of the worksheet) to the class. It works well to bring up an internet applet simulating the game (I'd recommend http://math.ucsd.edu/%7Ecrypto/Monty/monty.html) on the Smartboard (if available) and let a student come up and play a few rounds of the game so that everyone can see how it works. Explain to the students the two basic strategies a player could follow: either switch doors after the first pick to the one that wasn't opened or stay with the first pick even after you are shown one of the goat prizes. Ask the students what they think the best strategy is and why before they run the experiment. Then distribute the worksheets and break the class up unto groups of two. Each group should receive three cups and three prizes (1 car, 2 goats). Number the cups 1, 2, and 3 and have the students take turns playing the game with one student as the host and the other as the contestant and switching roles periodically. They should record in a table the number of wins and losses when the contestant switches doors and (separately) the number of wins and losses when the contestant doesn't switch. They should try each strategy 50 times. After they are finished, compile the class data and compute the winning percentages for the two different strategies. Ask the students to analyze the data and pick an optimal strategy according to the experimental probabilities. Then ask them to try to find the theoretical probability of winning for each strategy. It may help to draw a tree diagram or an area model here, but you may need to draw separate diagrams for each strategy.

Students may have a hard time understanding that the probability of winning can be other than 50/50 when there are two doors remaining. It may help them to understand if you change the problem this way: instead of picking from 3 doors, say there are 100 doors to pick from initially and only one contains a good prize. After the contestant chooses one, the host opens 98 (all but one) of the remaining doors leaving the chosen door (1% chance of containing the grand prize) and one of the others (99% chance of containing the grand prize). Then it may be clearer that the best strategy is to switch because it's very unlikely the contestant picked the winning door the first time.

An interesting modification to this problem is to say there are 4 doors initially and three rounds. First the contestant picks a door and then the host opens a losing door from among the other three. Then the contestant picks another (closed) door and the host opens a door from among the remaining two. Finally, the contestant picks a third time and gets whatever is behind that door. There are a handful of different strategies one could apply here. Trying to generalize the best solution to the 3-door version, students may think the best strategy is to switch doors every time. The probability of winning with this strategy is 5/8. The best strategy is actually to pick the *same* door the first two times and then switch when there's only two remaining. With this strategy you have a ¾ chance of winning.

Lesson Source: http://webpages.marshall.edu/~stevens13/

Instructional Mode: groups of 2

Date Given: December 11, 2007 **Estimated Time**: 60 minutes

Date Submitted to Algebra^3: December 19, 2007

Name: _______________________________ Period: _________________________

Let's Make a Deal!

The Monty Hall Game

Suppose you're on a game show, and you're given the choice of three doors: behind one door is the Grand Prize and behind the other two doors are goats. You pick a door, say Door 1, and the host (who knows what is behind each door) opens another door, say Door 2, revealing a goat. The host then offers you the opportunity to change your selection to Door 3. You know that the Grand Prize is either behind Door 1 or Door 3. Should you stick with your original choice or switch? Does it make any difference?

Task

You will experiment with the problem described in the introduction. The problem is known as "The Monty Hall Problem," named for the game show host of Let's Make a Deal. You will have 3 tasks.

Task 1: Find the experimental probability of winning when you stick with the first choice and the probability of winning when you switch choices. As you are collecting data to find the experimental probability, try to think about what the theoretical probability of winning would be if you switched doors and what it would be if you didn't switch.

Task 2: Argue whether or not you should switch doors based on your opinion, research, and experiment.

1. You will play the Monty Hall Game as an experiment to determine whether or not you win more often when you switch doors. What is your prediction of the outcome? Why?

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2. Play the Monty Hall Game. Record your results in the table below. Be sure to play 50 times WITH switching doors and 50 times WITHOUT switching doors.

STRATEGY	**Switch Doors**	**Don't Switch Doors**
WINS		
LOSSES		
WINNING PERCENTAGE		

3. Do your results display a difference in your chance of winning based on whether or not you switched doors? Explain.

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4. How do your results compare with your prediction? __________________________

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5. What is the probability of winning the Monty Hall Game? Explain.

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6. Do believe that you have a better chance of winning if you switch doors? Why?

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