Exponential Function Word Problems

Exponential growth is modelled by

$$y = y_0 e^{kt}$$

There are four variables, the initial amount, y_0, the time t, the growth factor k, and the current amount y. You should be comfortable with finding any one of these four, given the other three. You should also try to understand how changing any one of these affects the others. (For instance, if k is increased, and y_0 and t are both fixed, will y increase, decrease, stay the same, etc?)

1. $10,000 is invested at an annual interest rate of 5% compounded continuously.

  (a) How long will it take for this initial investment to double in value?

  (b) How long will it take for this initial investment to triple?

  (c) How long will it take for this initial investment to quadruple?

  (d) Can you see the pattern? How long to reach 5 times the initial amount? 7 times the initial amount? etc.

2. Repeat the previous problem, but this time with an initial investment of $500. Also, do it with an initial investment of $250,000. How do your answers change? Can you see why?

3. A recent college graduate decides he would like to have $20,000 in five years to make a down payment on a home.

  (a) How much money will he need to invest today in order to have $20,000 in five years, given that he can invest at an annual interest rate of 4% compounded continuously?

  (b) Suppose instead he can invest at an annual interest rate of 6%. How much does he need to invest? Will he need to make a larger or a smaller initial investment than in part (a)? (First think through this without calculations. Then find the exact answer. Be sure to check that these are consistent.)

  (c) Suppose the interest rate is 4% again, but now he would like to have the $20,000 in *four* years. How much does he need to invest? Will he need to make a larger or a smaller initial investment than in part (a)? (Again, first do this without calculations, then find the exact answer.)

  (d) Suppose the interest rate is 4% again, but now he would like to have $30,000 in five years. How much does he need to invest? Will he need to make a larger or a smaller initial investment than in part (a)? (Again, first do this without calculations, then find the exact answer.)

4. The half-life of caffeine is 5 hours. This means the amount of caffeine in your bloodstream is reduced by 50% every 5 hours. A grande French Roast has 330 milligrams of caffeine. Let $Q(t)$ denote the amount of caffeine in your system t hours after consuming your grande French Roast. For simplicity, assume the entire grande French Roast is consumed instantly.

(a) How many milligrams of caffeine will be in your system after 5 hours? After 10 hours? After 15 hours? (Think! This part should not require a lot of work.)

(b) $Q(t) = Q_0 e^{-kt}$. Find Q_0 and k.

(c) How many milligrams of caffeine will be in your system after 2 hours?

5. A bacteria culture triples in size every 7 hours. Three hours from now, the culture has $8,000$ bacteria. If $Q(t)$ denotes the number of bacteria, then $Q(t) = Q_0 e^{kt}$ for some number Q_0 and some number k.

(a) Determine Q_0 and k.

(b) How many bacteria are there at time $t = 0$?

(c) How many bacteria are there after ten hours? Do you see the "easy" way to solve this part?