MA 113 — Calculus I Fall 2012
Exam 1 18 September 2012

Name: _____

Section: _____

Last 4 digits of student ID #: _____

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problem(s) on the front page of this exam.

(1) Find the slope, the y-intercept, and the x-intercept of the line defined by

$$5y - 4x = -6$$

A) slope $= -4$, y-int $= -6$, x-int $= 5$

B) slope $= -4/5$, y-int $= -6/5$, x-int $= 3$

C) slope $= 4$, y-int $= -6/5$, x-int $= 2/3$

D) slope $= 4/5$, y-int $= -6/5$, x-int $= 3/2$

E) None of the above.

(2) A stone is tossed in the air from ground level with an initial velocity of 20 m/s. Its height at time t seconds is $h(t) = 20t - 4.9t^2$ meters. Compute the average velocity of the stone over the time interval $[1, 3.5]$.

A) -2.05 m/s

B) 10.2 m/s

C) 3.59 m/s

D) -20 m/s

E) None of the above.

Record the correct answer to the following problem(s) on the front page of this exam.

(3) Solve for x:

$$(\sqrt{5})^x = 78\,125\,.$$

A) 31 250

B) $\log_5(2)$

C) 14

D) 10

E) None of the above.

(4) The function

$$g(x) = \frac{x^3 + 3x^2 - 4x}{x^2 + x - 12}$$

might have one or more of the following types of discontinuities.

I) a removable discontinuity

II) an infinite discontinuity (or vertical asymptote)

III) a jump discontinuity

Which of these types of discontinuities does this function have?

A) I only

B) II only

C) III only

D) I and II only

E) II and III only

Record the correct answer to the following problem(s) on the front page of this exam.

(5) Let $f(x)$ be the function

$$f(x) = \begin{cases} x^2 - c & \text{for } x < 7 \\ 4x + 3c & \text{for } x \geq 7 \end{cases} .$$

Find the value of c that makes the function $f(x)$ continuous for all real numbers.

A) 49/3

B) 21/4

C) 21/2

D) 77/4

E) None of the above.

(6) Find the inverse f^{-1} if f is defined by

$$f(x) = \frac{x-1}{1+4x} .$$

A) $\dfrac{1+x}{1-x}$

B) $\dfrac{2}{1-4x}$

C) $\dfrac{1+x}{-3}$

D) $\dfrac{1+x}{1-4x}$

E) $\dfrac{1+4x}{x-1}$

Record the correct answer to the following problem(s) on the front page of this exam.

(7) Which of these is a correct evaluation of

$$\lim_{x\to 0} \frac{x^2}{\sin^2(4x)} \, ?$$

A)
$$\lim_{x\to 0} \frac{x^2}{\sin^2(4x)} = \lim_{x\to 0} \frac{x}{\sin(4x)} = \lim_{x\to 0} \frac{4x}{16\sin(4x)} = \frac{1}{16} \, .$$

B)
$$\lim_{x\to 0} \frac{x^2}{\sin^2(4x)} = \lim_{x\to 0} \frac{x}{\sin^2(4)} = 0 \, .$$

C)
$$\lim_{x\to 0} \frac{x^2}{\sin^2(4x)} = \lim_{x\to 0} \frac{x^2}{4\sin^2(x)} = \frac{1}{4} \, .$$

D)
$$\lim_{x\to 0} \frac{x^2}{\sin^2(4x)} = \lim_{x\to 0} \frac{16x^2}{16\sin^2(4x)} = \frac{1}{16} \lim_{x\to 0} \left(\frac{4x}{\sin(4x)} \right)^2 = \frac{1}{16} \, .$$

E) There is no correct evaluation. Since the limit of the denominator is zero, the limit does not exist.

Record the correct answer to the following problem(s) on the front page of this exam.

(8) Which of the following functions are continuous at the point $x = 3$?

A) $\dfrac{1}{x^2 - 9}$

B) $\dfrac{3x - 7}{\sin(\frac{2\pi x}{3})}$

C) $\dfrac{x - 3}{x^2 - 9}$

D) $\cos\left(\dfrac{x - 3}{\pi}\right)$

E) None of the above.

(9) Suppose that the function f is continuous for all real numbers and that $f(x) = \dfrac{x^2 - 4}{x + 2}$ when $x \neq -2$. Find the value of $f(-2)$.

A) -4

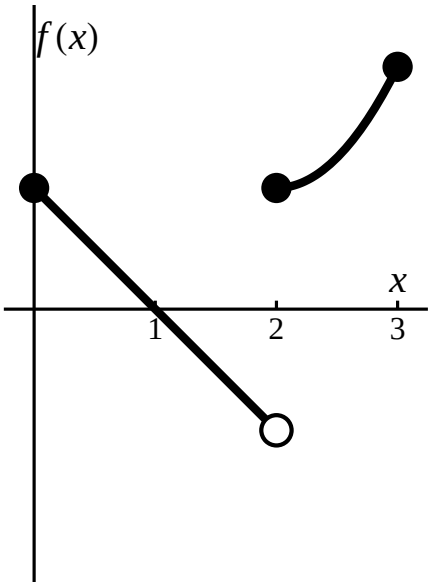
B) -2

C) -1

D) 0

E) 2

Record the correct answer to the following problem(s) on the front page of this exam.



(10) The figure above shows the graph of a function $f(x)$ with domain $0 \leq x \leq 3$. Which of the following statements are true?

I) $\lim_{x \to 2^-} f(x)$ exists

II) $\lim_{x \to 2^+} f(x)$ exists

III) $\lim_{x \to 2} f(x)$ exists

A) I only

B) II only

C) III only

D) I and II only

E) II and III only

Free Response Questions: Show your work!

(11) With an initial deposit of 200 dollars, the balance P in a bank account after t months is $P(t) = 200(1.09)^t$ dollars.

(a) Given a time interval $[a, b]$, what are the units of the average rate of change of the balance between time a and time b?

(b) Find the average rates of change over the intervals $[0.5, 0.7]$ and $[0.7, 0.9]$. *Show your work and include units.* You may leave your answer unsimplified, or you may provide an approximate answer from your calculator.

(c) Estimate numerically the instantaneous rate of change of the account balance at $t = 0.7$ months. Use at least one interval of length *less than* 0.2 to numerically estimate the instantaneous rate of change. *Show your work, clearly explain your process, and include units.*

(12) (a)True or False: If $f(x)$ and $g(x)$ are each discontinuous at $x = c$, then $f(x) + g(x)$ is always discontinuous at $x = c$.

If true, explain your answer. If false, provide a counterexample to the statement.

(b)True or False: The function $f(x)$ is continuous at $x = a$ if the left- and right-hand limits of $f(x)$ as $x \to a$ exist and are equal.

If true, explain your answer. If false, provide a counterexample to the statement.

(13) Evaluate the following limits using the limit laws, if the limits exist. If a limit does not exist, explain why it does not exist.

(a) $\lim_{x\to 2}\left[(x^2 - x + 1)2^x\right]$

(b) $\lim\limits_{x\to 3} \dfrac{x}{x^2 - 9}$

(14) Evaluate the following limit using the Squeeze Theorem. Show your work.

$$\lim_{x \to 0} \left[x^2 \cos\left(\frac{1}{x}\right) \right] .$$

(15) Evaluate the following limits using algebraic techniques, if the limits exist. If a limit does not exist, explain why it does not exist.

(a) $\lim_{x \to 8} \dfrac{\sqrt{x-4}-2}{x-8}$

(b) $\lim_{x \to 0} \dfrac{(1+2x)^2-1}{x}$