



**Record the correct answer to the following problems on the front page of this exam.**

1. Find the line that passes through the point  $(-2, 2)$  and is perpendicular to the line  $2x + y = 1$ .

(A)  $y = -2x - 2$

(B)  $y = \frac{1}{2}x + 3$

(C)  $y = 2x + 6$

(D)  $y = -\frac{1}{2}x + 1$

(E)  $y = -2x + 1$

2. Suppose that  $f$  is a function of the form  $f(x) = Ae^{kx}$ ,  $f(1) = 3$  and  $f(3) = 12$ . Find  $A$  and  $k$ .

(A)  $A = 3/2, k = 2$

(B)  $A = 3, k = 2$

(C)  $A = 6, k = 2$

(D)  $A = 3/2, k = \ln(2)$

(E)  $A = 3, k = \ln(2)$

3. Suppose that  $\cos(\theta) = x$  and  $\pi < \theta < 2\pi$ . Find  $\cos(-\theta)$  and  $\sin(-\theta)$ .

(A)  $\cos(-\theta) = \sqrt{1 - x^2}, \sin(-\theta) = x$

(B)  $\cos(-\theta) = -x, \sin(-\theta) = -\sqrt{1 - x^2}$

(C)  $\cos(-\theta) = x, \sin(-\theta) = -\sqrt{1 - x^2}$

(D)  $\cos(-\theta) = -x, \sin(-\theta) = \sqrt{1 - x^2}$

(E)  $\cos(-\theta) = x, \sin(-\theta) = \sqrt{1 - x^2}$

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4. If  $\lim_{x \rightarrow 3} f(x) = 2$  and  $\lim_{x \rightarrow 3} (f(x)g(x)) = -2$  find

$$\lim_{x \rightarrow 3} (2f(x) + g(x)).$$

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) 5

5. Suppose that a function  $f$  is defined by

$$f(x) = \begin{cases} 2x - 1, & x < 2 \\ c, & x = 2 \\ x, & x > 2 \end{cases}$$

and we know that  $f$  is left-continuous at 2. Find  $c$ .

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) There is not enough information to answer this question.

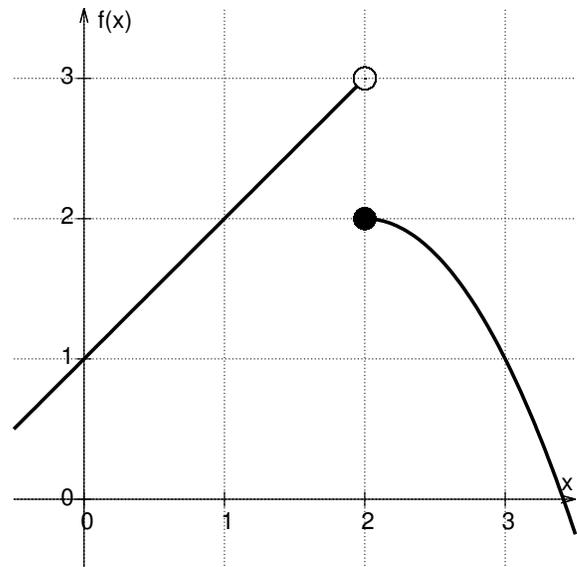
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6. Let the position of a particle be given by  $p(t) = t^2$ . Find the time  $a$  so that the average velocity of the particle for  $2 \leq t \leq a$  is equal to 6.

- (A) 2
- (B) 3
- (C) 4
- (D) 5
- (E) There is more than one correct answer.

7. Use the graph of  $f$  below to determine which of the following statements is true.

- (A)  $f$  is continuous at  $x = 2$
- (B)  $\lim_{x \rightarrow 2} f(x) = 2$
- (C)  $\lim_{x \rightarrow 2^+} f(x) = 3$
- (D)  $\lim_{x \rightarrow 2^-} f(x) = f(2)$
- (E)  $\lim_{x \rightarrow 2^-} f(x) = 3$



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8. Let  $f$  and  $g$  be functions that are defined on the real line. Four of the statements are true for any functions  $f$  and  $g$ . One of the statements may fail for some choices of  $f$  and  $g$ . Which of the statements may be false?

(A) If  $f(1) = g(1)$ , then  $\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} g(x)$ .

(B) If  $\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} g(x)$ , then  $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} g(x)$ .

(C) If  $\lim_{x \rightarrow 1} f(x) = 7$  and  $f$  is continuous at  $x = 1$ , then  $f(1) = 7$ .

(D) If  $\lim_{x \rightarrow 1} f(x) = 2$  and  $\lim_{x \rightarrow 1} g(x) = 3$ , then  $\lim_{x \rightarrow 1} (3f(x) - 2g(x)) = 0$ .

(E) If  $\lim_{x \rightarrow 1^+} g(x) = \lim_{x \rightarrow 1^-} g(x) = 2$ , then  $\lim_{x \rightarrow 1} g(x) = 2$ .

9. Suppose that  $f(x) = (x - 1)^2$ ,  $h(x) = 0$ ,  $g$  is a function with domain  $(-\infty, \infty)$ , and  $h(x) \leq g(x) \leq f(x)$  for all  $x$ . Then we can use the squeeze theorem to find

(A)  $\lim_{x \rightarrow 0} g(x) = 0$

(B)  $\lim_{x \rightarrow 0} g(x) = -1$

(C)  $\lim_{x \rightarrow 0} g(x) = 1$

(D)  $\lim_{x \rightarrow 1} g(x) = 0$

(E)  $\lim_{x \rightarrow 1} g(x) = 1$

10. Find the value of the limit  $\lim_{x \rightarrow 0} \frac{\tan(2x)}{x}$ .

(A)  $-2$

(B)  $-1$

(C)  $0$

(D)  $1$

(E)  $2$

**Free Response Questions: Show your work!**

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11. A rectangle has perimeter of 30 meters.
- (a) Make a sketch of the rectangle and label the sides in your sketch. Write a function  $A$  that gives the area of the rectangle as a function of the length of one of the sides.
  - (b) Give the domain of the function  $A$ . The sides of a rectangle cannot be of negative length, but we allow the cases where one of the sides is of length 0.

**Free Response Questions: Show your work!**

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12. Let  $f(x) = \frac{x+2}{x-3}$ . Find the inverse function  $f^{-1}$  and the domain and range of  $f^{-1}$ .

**Free Response Questions: Show your work!**

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13. Let  $f(x) = \frac{1}{2x - 1}$ .

- (a) Find the slope of the line that passes through the points  $(1, f(1))$  and  $(1 + h, f(1 + h))$ . Simplify your answer.
- (b) Take the limit as  $h$  tends to zero of the expression found in part a). Use the limit laws to justify your evaluation of the limit.
- (c) Give the geometric interpretation of your answer to part (b).

**Free Response Questions: Show your work!**

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14. For each limit below, find the limit if it exists or explain why it does not exist. Carefully justify your answers.

(a)  $\lim_{x \rightarrow 1} \frac{e^x(x-1)}{x^2-1}$ .

(b)  $\lim_{x \rightarrow 1} \frac{e^x(x+1)}{x^2-1}$ .

**Free Response Questions: Show your work!**

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15. (a) State the Intermediate Value Theorem.
- (b) Use the Intermediate Value Theorem to find an interval that contains a solution of the equation

$$x^3 + x + 1 = -3.$$