

Name: _____

Section and/or TA: _____

Do not remove this answer page. You will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or communication capabilities is permitted. You may not use a phone or other communication device during the exam.

The exam consists of 12 multiple choice questions that are worth 5 points each and 4 free response questions that are worth 10 points each. You should work the multiple choice questions on the question page. After you have checked your work carefully, record your answers by completely filling in the circle below that corresponds to your answer. If you must change your answer, make a note on the front of the exam. Be sure to check carefully when you transfer your answers to the cover sheet.

Show all work to receive full credit on the free response problems. You do not need to compute a decimal approximation to your answer. For example, the answer 4π is preferred to 12.57.

Multiple Choice Questions

1 A B C D E**7** A B C D E**2** A B C D E**8** A B C D E**3** A B C D E**9** A B C D E**4** A B C D E**10** A B C D E**5** A B C D E**11** A B C D E**6** A B C D E**12** A B C D E

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

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Multiple Choice Questions

1. (5 points) Let f be defined by $f(x) = 3x + 6$, find the inverse function $f^{-1}(x)$.

A. $f^{-1}(x) = \frac{1}{3x + 6}$

B. $f^{-1}(x) = \frac{1}{3}x - 6$

C. $f^{-1}(x) = \frac{1}{3}x - 2$

D. $f^{-1}(x) = -3x - 6$

E. $f^{-1}(x) = 3x - 2$

2. (5 points) A ball is thrown in the air so that its height h in meters at time t seconds is $h(t) = -5t^2 + 40t$. Find the average velocity on the interval $[5, 7]$.

A. -20 meters/second

B. 20 meters/second

C. 35 meters/second

D. -40 meters/second

E. 40 meters/second

3. (5 points) If $\lim_{x \rightarrow 3} f(x) = 5$, all but one of the following statements must be true. Select the statement that may be false.

A. $\lim_{x \rightarrow 3} xf(x) = 15$

B. $\lim_{x \rightarrow 3^-} f(x) = 5$

C. $\lim_{x \rightarrow 3} f^2(x) = 25$

D. $\lim_{x \rightarrow 3^+} f(x) = 5$

E. $f(3) = 5$

4. (5 points) Which answer best describes the behavior of $f(x) = \frac{1}{x-2}$ at $x = 2$.

A. $\lim_{x \rightarrow 2^+} \frac{1}{x-2} = -\infty$ and $\lim_{x \rightarrow 2^-} \frac{1}{x-2} = +\infty$

B. $\lim_{x \rightarrow 2} \frac{1}{x-2} = -\infty$

C. $f(2) = \infty$

D. $\lim_{x \rightarrow 2^+} \frac{1}{x-2} = +\infty$ and $\lim_{x \rightarrow 2^-} \frac{1}{x-2} = -\infty$

E. $\lim_{x \rightarrow 2} \frac{1}{x-2} = +\infty$

5. (5 points) Suppose that $\lim_{x \rightarrow 3} f(x) = -2$. Find $\lim_{x \rightarrow 3} (xf(x) + x^2)$.
A. 3 B. 0 C. 6 D. 13 E. 15

6. (5 points) Find the limit $\lim_{x \rightarrow a} \frac{x^2 - a^2}{x - a}$ or state that it does not exist.
A. $-2a$ B. $+\infty$ C. $2a$ D. 0 E. Does not exist

7. (5 points) Let $a > 0$. Find the limit $\lim_{h \rightarrow 0} \frac{\sqrt{a+2h} - \sqrt{a}}{h}$.

- A. $\frac{2}{\sqrt{2a}}$ B. $\frac{1}{\sqrt{a}}$ C. $\frac{2}{\sqrt{a}}$ D. $\frac{1}{\sqrt{2a}}$ E. $\frac{1}{2\sqrt{a}}$

8. (5 points) Suppose that $0 \leq f(x) \leq x^2 - 2x + 1$. There is one value a where we can use the squeeze theorem to find the limit $L = \lim_{x \rightarrow a} f(x)$. Find a and L .

- A. $a = 1, L = 1$
B. $a = 1, L = 0$
C. $a = -1, L = 0$
D. $a = 0, L = 0$
E. $a = 0, L = 1$

9. (5 points) Find $\lim_{x \rightarrow -\infty} \frac{|x - 2|}{x}$.

- A. $+\infty$ B. -1 C. $-\infty$ D. 1 E. 0

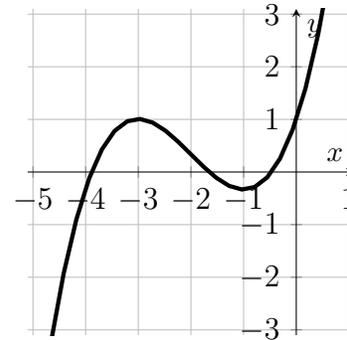
10. (5 points) If the tangent line to the graph of f at $x = 2$ is $y = 3x - 5$, give the value of the derivative $f'(2)$.

- A. 2 B. 1 C. -5 D. 4 E. 3

11. (5 points) Give the largest set where the function $f(x) = \sqrt{x-1}$ is continuous

- A. $[1, \infty)$
- B. $[0, 1]$
- C. $(-\infty, 1]$
- D. $(-\infty, 1) \cup (1, \infty)$
- E. $(-\infty, \infty)$

12. (5 points) The graph of f is shown at right. For which of the following values of x is the derivative $f'(x) < 0$?



- A. $x = -1$ B. $x = -3$ C. $x = -2$ D. $x = -4$ E. $x = 0$

Free response questions, show all work

13. (10 points) For each of the following limits, give the value of the limit if the value is a finite number, $+\infty$, or $-\infty$ or state that the limit does not exist. Use the results about limits discussed in class to justify your answers.

(a) $\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x - 2}$

(b) $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x - 2}$

(c) $\lim_{x \rightarrow 3} \frac{x^2 - 3x + 2}{x - 3}$

14. (10 points) Consider the function f defined by

$$f(x) = \begin{cases} x + 2, & x < 2 \\ a, & x = 2 \\ bx, & 2 < x \end{cases}$$

where a and b are constants.

- (a) Find the limits $\lim_{x \rightarrow 2^-} f(x)$ and $\lim_{x \rightarrow 2^+} f(x)$. Your answers may depend on a or b .
- (b) Find the values of a and b so that f is continuous for all x .

15. (10 points) (a) State the intermediate value theorem.
- (b) Find an interval $[a, b]$ which contains a solution of the equation $x^3 + x = 3$. Use the intermediate value theorem to show that the interval you found contains a solution to this equation.

16. (10 points) (a) State the definition of the derivative of a function f at a point a .
- (b) Use the definition to find the derivative $f'(3)$ for the function $f(x) = x^2 + x$.