MA 113 — Calculus I Spring 2013
Exam 1 (Solutions) February 5, 2013

Name: ____________________

Section: __________

Last 4 digits of student ID #: __________

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your final answers in the multiple choice answer box on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the multiple choice answer box.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer (unsupported answers may not receive credit).
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

Question					
1	A	B	C	**D**	E
2	A	**B**	C	D	E
3	**A**	B	C	D	E
4	A	B	**C**	D	E
5	A	**B**	C	D	E
6	A	B	C	D	**E**
7	A	B	C	**D**	E
8	**A**	B	C	D	E
9	A	B	**C**	D	E
10	A	B	C	**D**	E

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problem on the front page of this exam.

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(1) Find the slope, the y-intercept, and the x-intercept of the line defined by

$$2x + 3y = 1 \quad .$$

A) slope $= 2/3$, y-int $= 1/3$, x-int $= 1/2$

B) slope $= 2/3$, y-int $= 1/2$, x-int $= 1/3$

C) slope $= -2/3$, y-int $= 1/2$, x-int $= 1/3$

D) slope $= -2/3$, y-int $= 1/3$, x-int $= 1/2$

E) None of the above.

(2) A particle is moving along a straight line so that its position at time t seconds is given by $s(t) = 4t^2 - t$ meters. Compute the average velocity of the particle over the time interval $[2, 2.5]$.

A) 68 m/s

B) 17 m/s

C) -17 m/s

D) 16 m/s

E) None of the above.

Record the correct answer to the following problem on the front page of this exam.

(3) Solve for x:

$$2^{3x+2} = a, \qquad \text{where } a > 0.$$

A) $x = \frac{1}{3}(\log_2 a) - \frac{2}{3} = \frac{\ln a}{3 \ln 2} - \frac{2}{3}$

B) $x = \frac{2}{3}(\log_2 a) - \frac{1}{3} = \frac{2 \ln a}{3 \ln 2} - \frac{1}{3}$

C) $x = \frac{1}{2}(\log_2 a) - \frac{2}{3} = \frac{\ln a}{2 \ln 2} - \frac{2}{3}$

D) $x = \frac{1}{2}(\log_3 a) - \frac{2}{3} = \frac{\ln a}{2 \ln 3} - \frac{2}{3}$

E) None of the above.

(4) Find the inverse f^{-1} if f is defined by

$$f(x) = \frac{2x+1}{3x+2}.$$

A) $\frac{2x+1}{3x+2}$

B) $\frac{2x-1}{3x+2}$

C) $\frac{2x-1}{-3x+2}$

D) $\frac{3x+2}{2x-1}$

E) $\frac{3x+2}{2x+1}$

Record the correct answer to the following problem on the front page of this exam.

(5) Let $f(x)$ be a function such that

$$f(x) = \begin{cases} x^2 - c & \text{for } x < 2 \\ -4x + 3c & \text{for } x > 2 \end{cases}.$$

Find the value of c such that $\lim_{x\to 2} f(x)$ exists.

A) 1
B) 3
C) 6
D) -1
E) None of the above.

(6) Which of the following functions has a removable discontinuity at $x = 0$?

A) $f(x) = \dfrac{1}{x^2}$ for $x \neq 0$, and $f(0) = 0$.
B) $f(x) = x$ for $x \leq 0$, and $f(x) = 1 - x$ for $x > 0$.
C) $f(x) = \sin(\frac{1}{x})$ for $x \neq 0$, and $f(0) = 1$.
D) $f(x) = \cos(\frac{1}{x})$ for $x \neq 0$, and $f(0) = 0$.
E) $f(x) = \dfrac{x}{\sin(2x)}$ for $x \neq 0$, and $f(0) = 0$.

Record the correct answer to the following problem on the front page of this exam.

(7) Which of the following statements is NOT true?

A) If f is continuous at c, then $f(\sqrt[3]{x})$ is continuous at c.

B) The function $f(x) = \sqrt{x^2 + 2x + 2}$ is continuous at every real number.

C) If $\lim_{x \to c^-} f(x) = 1$ and f is continuous at c, then $f(c) = 1$.

D) If $\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x) = 1$, then f is continuous at c.

E) If $f(x)$ is a rational function and c is in the domain of f, then $\lim_{x \to c} f(x) = f(c)$.

(8) Suppose that $f(x) = x^2 - 4$ for $x < 4$, and $f(x) = x + 2$ for $x > 4$. Find the value of $f(4)$, if f is known to be left-continuous at 4.

A) 12

B) 6

C) 4

D) 0

E) None of the above.

Record the correct answer to the following problem on the front page of this exam.

(9) Which of the following statements is NOT true?

A) $\lim_{x \to -\infty} 2^x = 0$

B) $\lim_{x \to 2} \frac{1}{(x-2)^2} = \infty$

C) $\lim_{x \to 0^+} \frac{\sin x}{x} = \infty$

D) $\lim_{x \to 0^+} \ln x = -\infty$

E) $\lim_{x \to \infty} \frac{x^3 - 2x}{x^3 + 2x^2 - 1} = 1$

(10) Find the limit

$$\lim_{x \to 0} \frac{(2+x)^2 - 4}{x}.$$

A) 0

B) 1

C) 2

D) 4

E) The limit does not exist.

Free Response Questions: Show your work!

(11) With an initial deposit of 100 dollars, the balance P in a bank account after t years is $P(t) = 100(1.09)^t$ dollars.

(a)(2 pts.) Given a time interval $[a, b]$, what are the units of the average rate of change of the balance between time a and time b?

Solution:

Units=dollars/year

(b)(4 pts.) Find the average rates of change over the intervals $[1.7, 2]$ and $[2, 2.2]$. *Show your work and include units.* You may leave your answer unsimplified, or you may provide an approximate answer to two decimal places from your calculator.

Solutions:

The average rate of change over $[1.7, 2]$ **is** **(2 pts.)**

$$\frac{P(1.7) - P(2)}{1.7 - 2} = \frac{100(1.09)^{1.7} - 100(1.09)^2}{1.7 - 2} = 10.10755228 \approx 10.11 \textbf{ dollars/year.}$$

The average rate of change over $[2, 2.2]$ **is** **(2 pts.)**

$$\frac{P(2.2) - P(2)}{2.2 - 2} = \frac{100(1.09)^{2.2} - 100(1.09)^2}{2.2 - 2} = 10.32751659 \approx 10.33 \textbf{ dollars/year.}$$

(deduct a total of 1 point if correct units are not given)

(c)(4 pts.) Estimate numerically the instantaneous rate of change of the account balance at $t = 2$ years. Use two intervals - at least one of them has length *less than* 0.2 - to numerically estimate the instantaneous rate of change. *Show your work, clearly explain your process, and include units.*

Solution:

We choose two intervals: $[2, 2.2]$ **and** $[1.9, 2]$**. Using** $[2, 2.2]$**, we obtain**

$$\textbf{The instantaneous rate at } t = 2 \textbf{ years} \approx \frac{P(2.2) - P(2)}{2.2 - 2}$$
$$\approx 10.33 \quad \textbf{dollars/year}.$$

Using $[1.9, 2]$**, we obtain**

$$\textbf{The instananeous rate at } t = 2 \textbf{ years} \approx \frac{P(1.9) - P(2)}{1.9 - 2}$$
$$= \frac{100(1.09)^{1.9} - 100(1.09)^2}{1.9 - 2} \approx 10.19 \quad \textbf{dollars/year}.$$

(2 pts. each; deduct a total of 1 point if correct units are not given)

(12) Evaluate the following limits using the limit and continuity laws, if the limits exist. If a limit does not exist, explain why it does not exist.

(a)(5 pts.) $\lim_{x\to 4} 3^x\sqrt{2x^2+1}$

Solution:

$$\begin{aligned}\lim_{x\to 4} 3^x\sqrt{2x^2+1} &= \lim_{x\to 4} 3^x \cdot \lim_{x\to 4}\sqrt{2x^2+1}\\ &= 3^4\cdot\sqrt{2(4)^2+1}\\ &= 81\sqrt{33}.\end{aligned}$$

Correct answer with correct work: 5 pts.

Incorrect answer with partially correct work: 1 to 3 pts.

Correct answer with no work: 2 pts.

(b)(5pts.) $\lim_{x\to\infty} \frac{\sqrt{4x^4+5x+100}}{5x^2-4x-10}$

Solution:

$$\begin{aligned}\lim_{x\to\infty} \frac{\sqrt{4x^2+5x+100}}{5x^2+4x+10} &= \lim_{x\to\infty} \frac{\frac{\sqrt{4x^2+5x+100}}{x^2}}{\frac{5x^2+4x+10}{x^2}}\\ &= \lim_{x\to\infty} \frac{\sqrt{4+\frac{5}{x^3}+\frac{100}{x^4}}}{5+\frac{4}{x}+\frac{10}{x^2}}\\ &= \frac{\sqrt{4}}{5}\\ &= \frac{2}{5}.\end{aligned}$$

Correct answer with correct work: 5 pts.

Incorrect answer with partially correct work: 1 to 3 pts.

Correct answer with no work: 2 pts.

(13) (10 pts.) Evaluate the following limit using the Squeeze Theorem. Show your work.

$$\lim_{x\to 1}(x-1)^2 \sin\left(\frac{1}{x-1}\right)$$

Solution: Since

$$-1 \leq \sin\left(\frac{1}{x-1}\right) \leq 1 \qquad \textbf{(1 pts.)}$$

for $x \neq 1$**, and** $(x-1)^2 \geq 0$**, we have**

$$-(x-1)^2 \leq (x-1)^2 \sin\left(\frac{1}{x-1}\right) \leq (x-1)^2 \qquad \textbf{(4 pts.)}$$

for any $x \neq 1$**. Note that**

$$\lim_{x\to 1}(x-1)^2 = 0 \qquad \textbf{and} \qquad \lim_{x\to 1}\{-(x-1)^2\} = 0. \qquad \textbf{(2pts).}$$

By the Squeeze Theorem, we obtain **(1pts.)**

$$\lim_{x\to 1}(x-1)^2 \sin\left(\frac{1}{x-1}\right) = 0. \qquad \textbf{(2 pts.)}$$

(14) (10 pts.) Evaluate the limit

$$\lim_{x\to 2} \frac{x^2-5x+6}{x^2-x-2},$$

using algebraic techniques, if the limit exists. If the limit does not exist, explain why it does not exist.

Solution:

$$\begin{aligned}
\lim_{x\to 2} \frac{x^2-5x+6}{x^2-x-2} &= \lim_{x\to 2} \frac{(x-2)(x-3)}{(x-2)(x+1)} \\
&= \lim_{x\to 2} \frac{x-3}{x+1} \\
&= \frac{2-3}{2+1} \\
&= -\frac{1}{3}.
\end{aligned}$$

Correct answer with correct work: 10 pts.
Incorrect answer with partially correct work: 3 to 7 pts.
Correct answer with no work: 3 pts.

(15) (10 pts.) Evaluate the limit

$$\lim_{x\to 2} \frac{x-2}{\sqrt{x-1}-1},$$

using algebraic techniques, if the limit exists. If the limit does not exist, explain why it does not exist.

Solution:

$$\begin{aligned}
\lim_{x\to 2} \frac{x-2}{\sqrt{x-1}-1} &= \lim_{x\to 2} \frac{(x-2)(\sqrt{x-1}+1)}{(\sqrt{x-1}-1)(\sqrt{x-1}+1)} \\
&= \lim_{x\to 2} \frac{(x-2)(\sqrt{x-1}+1)}{(\sqrt{x-1})^2-1^2} \\
&= \lim_{x\to 2} \frac{(x-2)(\sqrt{x-1}+1)}{x-2} \\
&= \lim_{x\to 2} (\sqrt{x-1}+1) \\
&= \sqrt{1}+1 \\
&= 2.
\end{aligned}$$

Correct answer with correct work: 10 pts.
Incorrect answer with partially correct work: 3 to 7 pts.
Correct answer with no work: 3 pts.