MA 113 Calculus I Spring 2014
Exam 1 Tuesday, 11 February 2014

Name: ______________________________

Section: ____________________________

Last 4 digits of student ID #: ________

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box.*

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

[C,E,D,E,B, A,C,A,C,A]

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. Let $f(x) = \sqrt{x-1}$ and $g(x) = x^2$. Find the domain of the composite function $(f \circ g) = f(g(x))$.

   (A) $[-1, 1]$

   (B) $\mathbb{R} = (-\infty, \infty)$

   (C) $(-\infty, -1] \cup [1, \infty)$

   (D) $[1, \infty)$

   (E) $(-1, 1)$

2. Find the equation of the line parallel to $y = 5x - 2$ passing through the point $(-1, 1)$.

   (A) $y = -5x - 2$

   (B) $y = 5x - 1$

   (C) $y = -\frac{1}{5}x - 4$

   (D) $y = \frac{1}{5}x - 2$

   (E) $y = 5x + 6$

Record the correct answer to the following problems on the front page of this exam.

3. Find the limit

$$\lim_{x \to a} \frac{x^2 + (1-a)x - a}{x - a}.$$

(A) 0

(B) 2

(C) a

(D) $a + 1$

(E) $1 - a$

4. Solve for x when $e^{4(t+x)} = 2e^{4t}$.

(A) $x = \ln 4$

(B) $x = \ln 2$

(C) $x = 2$

(D) $x = 4$

(E) $x = \dfrac{\ln 2}{4}$

Record the correct answer to the following problems on the front page of this exam.

5. Simplify $\sin(\arccos(x))$. The function arccos is also denoted by $\cos^{-1}$.

   (A) π

   (B) $\sqrt{1-x^2}$

   (C) x

   (D) 0

   (E) $\dfrac{1}{x}$

6. Suppose that a function f is defined by

$$f(x) = \begin{cases} x^2 - c & x \leq 1 \\ cx + 2 & x > 1. \end{cases}$$

   Find c such that $\lim_{x \to 1} f(x)$ exists.

   (A) $-\dfrac{1}{2}$

   (B) 4

   (C) 0

   (D) -2

   (E) There is not enough information to answer this question.

Record the correct answer to the following problems on the front page of this exam.

7. Suppose that f is a function defined for every real number and satisfies the following properties:

$$\lim_{x\to 2} f(x) = 1, \quad f(2) = 1, \quad \lim_{x\to 3^-} f(x) = 0, \quad \lim_{x\to 3^+} f(x) = -2, \quad \text{and} \quad f(3) = 0.$$

Which of the following statements is NOT true?

(A) f is continuous at $x = 2$

(B) f is left-continuous at $x = 3$

(C) $\lim_{x\to 3} f(x) = 0$

(D) $\lim_{x\to 2^+} f(x) = 1$

(E) $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x)$

8. Let $f(x) = e^{x^2}$. Find the slope of the secant line that passes through $(0, f(0))$ and $(2, f(2))$.

(A) $\frac{1}{2}(e^4 - 1)$

(B) $1 - e^4$

(C) 1

(D) -3

(E) $\ln 4$

Record the correct answer to the following problems on the front page of this exam.

9. Find the limit

$$\lim_{x \to 0} \frac{\sin 2x}{x}.$$

   (A) 0
   (B) The limit does not exist.
   (C) 2
   (D) $\frac{1}{2}$
   (E) 1

10. A rectangle has width w and length ℓ in meters. If the perimeter of the rectangle is fixed at 14 meters, find the function that describes the area of the rectangle as a function of w.

   (A) $7w - w^2$
   (B) $14w$
   (C) w^4
   (D) 14ℓ
   (E) $14w - 2w^2$

Free Response Questions: Show your work!

11. For each limit below, find the limit if it exists or explain why it does not exist. Carefully justify your answers.

    (a) $\lim\limits_{x\to 0} \dfrac{\ln(1-x)}{1+x}$

    (b) $\lim\limits_{t\to 1} \left(\dfrac{1}{t-1} - \dfrac{1}{t^2-3t+2}\right)$

(a) The function $\frac{\ln(1-x)}{1+x}$ is continuous at $x=0$, thus, we can evaluate the limit by substituting $x=0$ and obtain $\lim\limits_{x\to 0} \dfrac{\ln(1-x)}{1+x} = 0$.	Use of continuity (2 points)
	Value (3 points)
(b) We simplify the expression $\frac{1}{t-1} - \frac{1}{t^2-3t+2} = \frac{t-3}{t-2} \cdot \frac{1}{t-1}$	(2 points)
Next we observe that $\lim_{t\to 1} \frac{t-3}{t-2} = 2$ and $\lim_{t\to 1^\pm} \frac{1}{t-1} = \pm\infty$. Thus $\lim_{t\to 1\pm} \frac{t-3}{t-2} \cdot \frac{1}{t-1} = \pm\infty$.	Behavior near 1 (2 points)
Since the one-sided limits are infinite, the limit does not exist	Answer (1 point)
	Other explanations possible. Give 1 point for numerical evidence.

12. (a) State the squeeze theorem.

    (b) Evaluate the following limit. Justify your work.

$$\lim_{x\to 0} x^2 \sin(\frac{\pi}{x})$$

(a) Suppose that for x in an open interval containing c, except possibly at c, we have $$f(x) \leq g(x) \leq h(x)$$ and $$\lim_{x\to x} f(x) = \lim_{x\to x} h(x) = L.$$	$f(x) \leq g(x) \leq h(x)$ (1 point) $\lim f(x) = \lim h(x) = L$ (2 points)
Then $\lim_{x\to c} g(x)$ exists and $\lim_{x\to c} g(x) = L$.	Conclude that $\lim g(x) = L$ (2 points)
(b) Since $-1 \leq \sin(\pi/x) \leq 1$, we have $$-x^2 \leq x^2 \sin(\pi/x) \leq x^2$$	(2 points)
We have $\lim_{x\to 0} x^2 = \lim_{x\to 0} -x^2 = 0$.	(1 point)
Since the hypotheses of the squeeze theorem are satisfied, we have the conclusion that $\lim_{x\to 0} x^2 \sin(\pi/x) = 0$	Answer (2 points)

Free Response Questions: Show your work!

13. An object is thrown straight into the air from the surface of a distant planet. The height of the object in meters after t seconds is given by $h(t) = 4t - t^2$.

    (a) Find the average velocity of the object over the time interval $[1.4, 1.6]$. Include units in your answer.

    (b) Find the average velocity of the object over the time interval $[3, 3+k]$. Include units in your answer.

    (c) Compute the instantaneous velocity of the object at $t = 3$ by taking the limit as k tends to 0 of the expression found in part (b). Use limit laws to justify your answer.

(a) The average velocity is $\frac{h(1.6)-h(1.4)}{1.6-1.4} = 1$ meter/second	Method (1 point), Answer (1 point), Units (1 point)
(b) The average velocity on the interval $[3, 3+k]$ is $\frac{h(3+k)-h(3)}{k} = -2 - k$ meters/second	Method (2 points), Simplify (2 point), Deduct one point for missing units, unless a point was deducted for missing units in part (a).
(c) $\lim_{k\to 0}(-2-k) = -2$. Since $-2-k$ is continuous -or- using the rule for the limit of a sum and the basic limits, $\lim_{k\to 0} k = 0$ and $\lim_{k\to 0} -2 = -2$.	Value (2 points), justification (1 point)

14. Let $f(x) = \dfrac{4x-1}{2x+3}$. Find the inverse function f^{-1} and the domain and range of f^{-1}.

We solve $y = \frac{4x-1}{2x+3}$ to express x in terms of y and obtain $x = \frac{1+3y}{4-2y}$.	Set up to solve for x in terms of y (1 point) Correct solution for x (3 points), deduct one point for each error in algebra
Thus $f^{-1}(x) = \frac{1+3x}{4-2x}$	Answer (2 points)
Domain of f^{-1} is $\{x : x \neq 2\} = (-\infty, 2) \cup (2, \infty)$.	Domain (2 points)
Range of f^{-1} is $\{x : x \neq -3/2\} = (-\infty, -3/2) \cup (-3/2, \infty)$.	Range (2 points)
	Give credit if domain and range correspond to an incorrect answer for the inverse function.

Free Response Questions: Show your work!

15. Using the graph of the function f below, complete the following. If a function value or limit does not exist, write DNE.

(a) $\lim_{x \to -2^+} f(x) =$ ____________ 1 One point for each part

(b) $\lim_{x \to -2^-} f(x) =$ ____________ -1

(c) $\lim_{x \to -2} f(x) =$ ____________ Does not exist

(d) $f(-2) =$ ____________ 1

(e) Is f right-continuous at -2? ____________ Yes

(f) $\lim_{x \to 1^-} f(x) =$ ____________ 1

(g) $\lim_{x \to 1^+} f(x) =$ ____________ 1

(h) $\lim_{x \to 1} f(x) =$ ____________ 1

(i) $f(1) =$ ____________ 3

(j) Is f continuous at 1? ____________ No