MA 113 Calculus I   Spring 2015
Exam 1   Tuesday, 10 February 2015

Name: ——————————————

Section: ——————————————

Last 4 digits of student ID #: ————

This exam has ten multiple choice questions
(five points each) and five free response ques-
tions (ten points each). Additional blank sheets
are available if necessary for scratch work. No
books or notes may be used. Turn off your
cell phones and do not wear ear-plugs during
the exam. You may use a calculator, but not
one which has symbolic manipulation capa-
bilities.

On the multiple choice problems:

1. You must give your *final answers* in the
    multiple choice answer box on the front
    page of your exam.

2. Carefully check your answers. No credit
    will be given for answers other than those
    indicated on the *multiple choice answer
    box.*

On the free response problems:

1. Clearly indicate your answer and the rea-
    soning used to arrive at that answer *(un-
    supported answers may not receive credit)*,

2. Give exact answers, rather than decimal
    approximations to the answer (unless oth-
    erwise stated).

Each free response question is followed by
space to write your answer. Please write your
solutions neatly in the space below the ques-
tion. You are not expected to write your so-
lution next to the statement of the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

D, C, B, A, C   B, A, B, E, E

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. Find the equation of the line with y-intercept 4 and parallel to the line given by the equation $5x + 6y = 10$.

   (A) $y = -\frac{6}{5}x + \frac{25}{6}$

   (B) $y = -\frac{6}{5}x + 4$

   (C) $y = -\frac{5}{6}x + \frac{24}{5}$

   (D) $y = -\frac{5}{6}x + 4$

   (E) None of the above

2. Let $f(x) = \dfrac{5x - 2}{4x + 3}$. Which value of x is not in the domain of $f^{-1}(x)$?
   (First find a formula for $f^{-1}(x)$.)

   (A) 3/4

   (B) −3/4

   (C) 5/4

   (D) 2/5

   (E) None of the above.

3. Find $\lim\limits_{x \to c} e^{4x+1} + \lim\limits_{x \to 3} \dfrac{x^2 - 9}{x - 3}$.

   (A) $e^{4c} + 7$

   (B) $e^{4c+1} + 6$

   (C) $e^{4c+1} - 6$

   (D) $e^{4c} + e + 6$

   (E) $e^{4c+1} + c + 6$

4. If $\lim\limits_{x \to 3} f(x) = 5$ and $\lim\limits_{x \to 3}(f(x) - g(x)) = -8$ find

$$\lim_{x \to 3}(3f(x) + g(x)^2).$$

(A) 184

(B) 186

(C) 24

(D) 26

(E) 79

5. Suppose that a function f is defined by

$$f(x) = \begin{cases} 4x + 3, & 0 < x < 5 \\ c, & x = 5 \\ 2\sqrt{x + 4} + 1, & x > 5 \end{cases}$$

For what choice of c is f right-continuous at $x = 5$?

(A) 5

(B) 6

(C) 7

(D) 16

(E) 23

6. Suppose that a function f is defined by

$$f(x) = \begin{cases} 4x + 3, & 0 < x < 2 \\ 14, & x = 2 \\ x^2 - 3x + 5, & x > 2 \end{cases}$$

Find $\lim\limits_{x \to 2^-} f(x)$

(A) 3

(B) 11

(C) 14

(D) Cannot be determined with the given information

(E) None of the above

7. For which of the examples below can the Intermediate Value Theorem (IVT) be used to conclude that the equation has a solution lying in the given interval?

   I. $x^3 - x = 3$, $[0, 2]$

  II. $x^2 - x = 3$, $[4, 8]$

 III. $\dfrac{1}{x} - \dfrac{1}{2} = 0$, $[-1, 1]$

(A) I only

(B) II only

(C) I and II only

(D) I and III only

(E) II and III only

8. For a real number a, find $\lim\limits_{x \to a} \dfrac{x^2 + (2-a)x - 2a}{x - a}$. (Carefully factor the numerator.)

   (A) 2

   (B) $a + 2$

   (C) 0

   (D) $2 - a$

   (E) The limit does not exist.

9. Suppose that $f(x) = 4$, $h(x) = 4 - (x-3)^2$, $g(x)$ is a function with domain $(-\infty, \infty)$, and $h(x) \le g(x) \le f(x)$ for all x. Then we can use the squeeze theorem to conclude the following.

   (A) $\lim\limits_{x \to 0} g(x) = -5$

   (B) $\lim\limits_{x \to 0} g(x) = 4$

   (C) $\lim\limits_{x \to 3} g(x) = -5$

   (D) $\lim\limits_{x \to 3} g(x) = 3$

   (E) $\lim\limits_{x \to 3} g(x) = 4$

10. Find the average rate of change of the function $f(x) = \ln(x^2 + 4)$ between the points $(1, f(1))$ and $(4, f(4))$.

    (A) $\frac{1}{3}\ln(15)$

    (B) $\frac{\ln(20) + \ln(4)}{3}$

    (C) 0

    (D) $\frac{\ln(20) - \ln(5)}{5}$

    (E) $\frac{1}{3}\ln(4)$

11. (a) Find all solutions to the equation $\ln(18 - 5x) - 2\ln(x) = \ln(2)$.

$\ln(18 - 5x) - \ln(x^2) = \ln(2)$, where $x > 0$,
$\ln(\frac{18-5x}{x^2}) = \ln(2)$,
$\frac{18-5x}{x^2} = 2$, $2x^2 + 5x - 18 = 0$,
$(2x + 9)(x - 2) = 0$, $x = 2$ because $x > 0$.

(b) Suppose that the domain of $f(x)$ is $[4, 12]$ and that the range of $f(x)$ is $[-5, 8]$. Find the domain and range of $f(x - 4) + 10$.

The domain of $f(x - 4) + 10$ is given by the condition $4 \le x - 4 \le 12$. Thus the domain of $f(x - 4) + 10$ is given by $8 \le x \le 16$. The range of $f(x - 4) + 10$ is given by $-5 + 10 \le y \le 8 + 10$, so the range of $f(x - 4) + 10$ is $5 \le y \le 18$.

12. Let $f(x) = (x-4)^2 + 12$.

   (a) Use the graph of $f(x)$ to find the largest value of c such that $f(x)$ is one to one on the interval $(-\infty, c]$.

   $c = 4$. This corresponds to the left half of the graph of $f(x) = (x-4)^2 + 12$, which is a parabola.

   (b) Restrict the domain of f to $(-\infty, c]$, where c is the value found in part (a). Let g be the inverse of the function f. Find a formula for g and find the domain and range of g.

   $x = (y-4)^2 + 12$, $y - 4 = \pm\sqrt{x-12}$, $y = 4 \pm \sqrt{x-12}$. Thus
   $g(x) = y = 4 - \sqrt{x-12}$. The domain of $g(x)$ is $x \geq 12$.
   The range of $g(x)$ is $(-\infty, 4]$.

13. Let $f(x) = 2x^2 - 5x$.

    (a) Find the slope of the line that passes through the points $(3, f(3))$ and $(3 + h, f(3 + h))$.

    $$\frac{f(3 + h) - f(3)}{h} = \frac{[2(3 + h)^2 - 5(3 + h)] - [2 \cdot 3^2 - 5 \cdot 3]}{h}$$
    $$= \frac{12h + 2h^2 - 5h}{h} = \frac{7h + 2h^2}{h} = 7 + 2h.$$

    (b) Take the limit as h tends to zero of the expression found in part (a). Use the limit laws to justify your evaluation of the limit.

    $\lim_{h \to 0}(7 + 2h) = 7$.

    (c) Give a geometric interpretation in terms of the graph of $f(x)$ for your answer to part (b).

    The slope of the tangent line to the graph at $(3, f(3))$ is 7.

14. For each limit below, find the limit if it exists or explain why it does not exist. Carefully justify your answers.

(a) $\displaystyle\lim_{x\to 7}\frac{x^3-49x}{x-7}$

$$\lim_{x\to 7}\frac{x(x-7)(x+7)}{x-7}=\lim_{x\to 7}x(x+7)=7\cdot 14=98.$$

(b) $\displaystyle\lim_{x\to 8}\frac{\sqrt{x-4}-2}{x-8}$

$$\frac{\sqrt{x-4}-2}{x-8}\cdot\frac{\sqrt{x-4}+2}{\sqrt{x-4}+2}=\frac{x-8}{(x-8)(\sqrt{x-4}+2)}=\frac{1}{\sqrt{x-4}+2}.\text{ Thus}$$
$$\lim_{x\to 8}\frac{1}{\sqrt{x-4}+2}=\frac{1}{\sqrt{4}+2}=\frac{1}{4}.$$

15. For each limit below, find the limit if it exists or explain why it does not exist. Carefully justify your answers.

(a) $\displaystyle\lim_{h\to 0} \frac{\tan(3h)}{4h}$

$\dfrac{\tan(3h)}{4h} = \dfrac{\sin(3h)}{3h} \cdot \dfrac{1}{\cos(3h)} \cdot \dfrac{3}{4}$. Thus

$\displaystyle\lim_{h\to 0} \frac{\tan(3h)}{4h} = 1 \cdot 1 \cdot \frac{3}{4} = \frac{3}{4}$.

(b) $\displaystyle\lim_{\theta\to 0} \frac{\sec(\theta) - 1}{\theta}$

$\dfrac{\sec(\theta) - 1}{\theta} = \dfrac{1 - \cos(\theta)}{\theta} \cdot \dfrac{1}{\cos(\theta)}$. Thus $\displaystyle\lim_{\theta\to 0} \frac{\sec(\theta) - 1}{\theta} = 0 \cdot 1 = 0$.

(Recall that $\dfrac{1 - \cos(\theta)}{\theta} = \dfrac{1 - \cos^2(\theta)}{\theta} \cdot \dfrac{1}{1 + \cos(\theta)} = \dfrac{\sin(\theta)}{\theta} \cdot \dfrac{\sin(\theta)}{1 + \cos(\theta)}$.

Thus $\displaystyle\lim_{\theta\to 0} \frac{1 - \cos(\theta)}{\theta} = 1 \cdot 0 = 0$.)