MA 113 Calculus I Spring 2018
Exam 1 Tuesday, 6 February 2018

Name: _____

Section: _____

Last 4 digits of student ID #: _____

This exam has 12 multiple choice questions (five points each) and 4 free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-buds during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E

B, E, D, C, D, B, C, D, C, D, A, D

Exam Scores

Question	Score	Total
MC		60
13		10
14		10
15		10
16		10
Total		100

1. Solve the equation $\sin^{-1}(5x-3) = \dfrac{\pi}{3}$.

   (A) $\frac{6-\sqrt{3}}{10}$

   (B) $\frac{6+\sqrt{3}}{10}$

   (C) $\frac{7}{10}$

   (D) $\frac{3}{5}$

   (E) None of the above

2. Suppose that $f(x) = \begin{cases} \sqrt{x+3} & x < 6 \\ c & x = 6 \\ x^2 - 5x & x > 6. \end{cases}$

   What value of c makes f right continuous at $x = 6$?

   (A) 2

   (B) 3

   (C) 4

   (D) 5

   (E) 6

Record the correct answer to the following problems on the front page of this exam.

3. Find the domain of $f(x) = \sqrt{\ln(2x+5)}$.

   (A) $x \geq \frac{e-5}{2}$
   (B) $x \geq -\frac{5}{2}$
   (C) $x > -\frac{5}{2}$
   (D) $x \geq -2$
   (E) $x > 0$

4. Suppose that $f(x) = \begin{cases} \sqrt{x+3} & \text{if } x < 6 \\ c & \text{if } x = 6 \\ x^2 - 5x & \text{if } x > 6. \end{cases}$

   Find $\lim_{x \to 6^-} f(x)$.

   (A) 1
   (B) 2
   (C) 3
   (D) 4
   (E) 5

Record the correct answer to the following problems on the front page of this exam.

5. Let $s(t) = 3t^2 - 4t$ denote the distance measured in meters that an object moves after t seconds. Find the average velocity of the object between t and $t + h$ seconds.

   (A) $6t - 4 - h$ m/s

   (B) $6t + 4 - h$ m/s

   (C) $6t - 4 - 3h$ m/s

   (D) $6t - 4 + 3h$ m/s

   (E) $6t + 4 + 3h$ m/s

6. Suppose that $f(x)$ is continuous at all real numbers. Assume that $f(2) = f(5) = 0$ and that $f(x) \neq 0$ for all values $x \neq 2, 5$. Assume also that $f(-1) = 3$, $f(3) = -2$, and $f(7) = 4$. Use the Intermediate Value Theorem to determine which of the following statements might not be true.

   (A) The equation $f(x) = 2$ has at least two solutions.

   (B) The equation $f(x) = -1$ has exactly two solutions.

   (C) $f(x) > 0$ for all $x > 5$.

   (D) $f(x) > 0$ for all $x < 2$.

   (E) $f(x) < 0$ for all x satisfying $2 < x < 5$.

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7. Let a be a solution to the equation $\ln(x-2)+\ln(x-3)=\ln(2)$. Find a^3+a.

   (A) 36

   (B) 52

   (C) 68

   (D) 92

   (E) Cannot be determined

8. Suppose that $f(x)=\begin{cases} 2x+3, & \text{if } x<1 \\ 4x+1 & \text{if } 1\le x<4 \\ 3x-6 & \text{if } x\ge 4. \end{cases}$

   Compute $\lim\limits_{x\to 1^-} f(x)+\lim\limits_{x\to 1^+} f(x)+\lim\limits_{x\to 4^-} f(x)+\lim\limits_{x\to 4^+} f(x)$.

   (A) 30

   (B) 31

   (C) 32

   (D) 33

   (E) 34

Record the correct answer to the following problems on the front page of this exam.

9. Compute $\lim_{x\to 0^-} -3\frac{x}{|x|} + 7 \lim_{x\to 0^+} \frac{x}{|x|}$

   (A) 4

   (B) -4

   (C) 10

   (D) -10

   (E) None of the above

10. Assume that f, g, h are continuous functions at all real numbers. Assume that $\lim_{x\to 3} g(x) = 5$ and $\lim_{x\to 5} f(x) = 4$. Find $3 \lim_{x\to 3} f(g(x)) + 2 \lim_{x\to 3} g(x)^2$.

    (A) 56

    (B) 58

    (C) 60

    (D) 62

    (E) 64

Record the correct answer to the following problems on the front page of this exam.

11. Find $\lim_{x\to\infty} \dfrac{\sqrt{4x^2+5x-2}}{4x-2}$.

(A) $\frac{1}{2}$

(B) 1

(C) $-\frac{1}{2}$

(D) -1

(E) ∞

12. Which of the following functions is continuous at $x = 4$?

(A) $\dfrac{1}{x^2-16}$

(B) $\dfrac{4x+6}{\sin(\frac{\pi x}{2})}$

(C) $\dfrac{x-4}{x^2-16}$

(D) $\sin(\dfrac{x-4}{2\pi})$

(E) None of the above

13. Find the limits or state that the limit does not exist. In each case, justify your answer.

    (Students who guess the answer based on a few values of the function will not receive full credit.)

    (a) $\lim_{t\to 2} \dfrac{\sqrt{4t+1}-3}{t-2}$

    $\lim_{t\to 2} \dfrac{\sqrt{4t+1}-3}{t-2} = \lim_{t\to 2} \dfrac{\sqrt{4t+1}-3}{t-2} \cdot \dfrac{\sqrt{4t+1}+3}{\sqrt{4t+1}+3} = \lim_{t\to 2} \dfrac{(4t+1)-9}{(t-2)(\sqrt{4t+1}+3)} =$
    $\lim_{t\to 2} \dfrac{4}{\sqrt{4t+1}+3} = \dfrac{4}{3+3} = \dfrac{2}{3}.$

    (b) $\lim_{x\to 2} \dfrac{(x-2)\ln(x+1)}{x^2-5x+6}$

    $\lim_{x\to 2} \dfrac{(x-2)\ln(x+1)}{x^2-5x+6} = \lim_{x\to 2} \dfrac{(x-2)\ln(x+1)}{(x-2)(x-3)} = \lim_{x\to 2} \dfrac{\ln(x+1)}{x-3} = \dfrac{\ln(3)}{-1} = -\ln(3)$

14. Find the limits or state that the limit does not exist. In each case, justify your answer.

    (Students who guess the answer based on a few values of the function will not receive full credit.)

    (a) $\lim_{x\to\infty} \dfrac{3x^4 - 2x + 7}{7x^4 + 5x^3}$

    $\lim_{x\to\infty} \dfrac{3x^4 - 2x + 7}{7x^4 + 5x^3} = \lim_{x\to\infty} \dfrac{3 - \frac{2}{x^3} + \frac{7}{x^4}}{7 + \frac{5}{x}} = \dfrac{3}{7}.$

    (b) $\lim_{x\to\frac{\pi}{6}} \dfrac{\tan(x + \frac{\pi}{12})}{\cos(x + \frac{\pi}{6})}$

    $\lim_{x\to\frac{\pi}{6}} \dfrac{\tan(x + \frac{\pi}{12})}{\cos(x + \frac{\pi}{6})} = \dfrac{\tan(\frac{\pi}{6} + \frac{\pi}{12})}{\cos(\frac{\pi}{6} + \frac{\pi}{6})} = \dfrac{\tan(\frac{\pi}{4})}{\cos(\frac{\pi}{3})} = \dfrac{1}{\frac{1}{2}} = 2.$

15.

(a) Suppose that $f(x) = Ce^{kx}$. If $f(0) = 12$ and $f(4) = 36$, then find C and k.

$C = f(0) = 12$. $36 = f(4) = 12e^{4k}$. Then $3 = e^{4k}$, $\ln(3) = 4k$, $k = \dfrac{\ln(3)}{4}$.

(b) Let $f(x) = x^5 + 3$. Find the inverse function $f^{-1}(x)$ and then compute $f^{-1}(2)$.

$y = x^5 + 3$, $x^5 = y - 3$, $x = \sqrt[5]{y-3}$. Thus $f^{-1}(x) = \sqrt[5]{x-3}$.

$f^{-1}(2) = \sqrt[5]{(-1)} = -1$.

16. Assume that the position of an object after t seconds is given by $f(t) = 2t^2 - 3t$ meters.

    (a) Write an expression for the average velocity of the object on the interval $[2, t]$. Include units!

    Average velocity $= \dfrac{f(t) - f(2)}{t - 2} = \dfrac{(2t^2 - 3t) - 2}{t - 2} = \dfrac{(2t + 1)(t - 2)}{t - 2} = 2t + 1$ meters per second.

    (b) Compute the average velocity over the time intervals $[2, 2.001]$ and $[1.999, 2]$ to estimate the instantaneous velocity. Include units!

    Average velocity over $[2, 2.001]$ is $2(2.001) + 1 = 5.002$ meters per second. Average velocity over $[1.999, 2]$ is $2(1.999) + 1 = 4.998$ meters per second.

    (c) Take the limit as t approaches 2 of the expression you found in part (a) to find the instantaneous velocity of the object at time $t = 2$ seconds. Include units!

    Instantaneous velocity at $t = 2$ is $\lim_{t \to 2}(2t + 1) = 5$ meters per second.