Exam 1

Form A

Name: ______________________ Section and/or TA: ____________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 14 multiple choice questions that count 5 points each and 3 free response questions that count 10 points each. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems.

Multiple Choice Questions

1	A	B	C	D	E	**8**	A	B	C	D	E
2	A	B	C	D	E	**9**	A	B	C	D	E
3	A	B	C	D	E	**10**	A	B	C	D	E
4	A	B	C	D	E	**11**	A	B	C	D	E
5	A	B	C	D	E	**12**	A	B	C	D	E
6	A	B	C	D	E	**13**	A	B	C	D	E
7	A	B	C	D	E	**14**	A	B	C	D	E

SCORE

Multiple Choice	17	18	19	Total Score
70	10	10	10	100

Trigonometric Identities

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$
$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$
$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

Multiple Choice Questions

1. Find $\lim_{x\to 2} \sqrt{x+23} + \lim_{x\to 4} \frac{x^2-16}{x-4}$.

   A. 15

   B. 12

   C. 14

   D. 16

   E. 13

2. If $\lim_{x\to 3} f(x) = 7$ and $\lim_{x\to 3} (f(x)g(x)) = 28$, then find

$$\lim_{x\to 3} \left(5f(x) + \sqrt{g(x)} + (g(x))^2\right).$$

   A. 54

   B. 53

   C. 56

   D. 55

   E. 52

3. Suppose that a function f is defined by

$$f(x) = \begin{cases} 3x - 2 & 0 < x < 4 \\ 18 & x = 4 \\ x^2 - 5x + 6 & x > 4 \end{cases}$$

Let $M = \lim_{x \to 4^-} f(x)$ and $N = \lim_{x \to 4^+} f(x)$. Find $3M + 5N$.

A. 12

B. 40

C. 56

D. 144

E. 18

4. The position function for a particle moving along a line is given by $s(t) = t^3 + 1$. Find the average velocity of the particle on the interval $[2, 5]$.

A. 38

B. 40

C. 37

D. 39

E. 36

5. Find all values of x which satisfy the given equation

$$\log_2(x) + \log_2(x-3) = 2.$$

A. $x = -1$

B. $x = 4$

C. $x = 4$ and $x = -1$

D. $x = 2$

E. The equation has no solutions.

6. Find all values of θ in the interval $[0, 2\pi]$ for which

$$\sin(\theta) = \cos(\theta).$$

A. $\theta = \frac{\pi}{4}$ **and** $\theta = \frac{5\pi}{4}$.

B. $\theta = 0, \theta = \pi$, and $\theta = 2\pi$.

C. $\theta = \frac{\pi}{4}, \theta = \frac{5\pi}{4}$ and $\theta = \frac{9\pi}{4}$.

D. $\theta = \frac{\pi}{3}$ and $\theta = \frac{\pi}{6}$

E. The equation has no solutions in the specified interval.

7. Consider the function defined by

$$f(x) = \begin{cases} c^2x + c & \text{if } x < 1 \\ x^2 - 2x + 3 & \text{if } x \geq 1 \end{cases}.$$

For which value(s) of c is this function continuous?

A. $c = 1$

B. $c = 3$

C. $c = 2$

D. $c = 1$ and $c = -2$

E. This function is not continuous for any value of c.

8. Evaluate the limit

$$\lim_{x\to\infty} \frac{\sqrt{9x^4 + 5x + 1}}{4x^2 + 3}$$

A. $\frac{3}{4}$

B. $\frac{\sqrt{15}}{7}$

C. $\frac{9}{4}$

D. ∞

E. $-\infty$

9. Find all of the vertical asymptote(s) of the function

$$g(x) = \frac{x^2 + x}{x^3 - x}$$

   A. $x = 0$
   B. $x = 1$ and $x = 0$
   C. $x = 1$
   D. $x = 1,\ x = -1$, and $x = 0$
   E. $x = 1$

10. If $-x^2 - 2x \leq f(x) \leq x^2 + 2x + 2$, for all x, find $\lim_{x \to -1} f(x)$.
   A. $-1/8$
   B. 1
   C. $-1/16$
   D. 8
   E. Does not exist

11. If a function $f(x)$ is not defined at $x = a$, which of the following is a true statement?

A. $\lim_{x \to a} f(x)$ cannot exist.

B. $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$

C. $\lim_{x \to a} f(x)$ must approach infinity.

D. $\lim_{x \to a} f(x)$ might be equal to zero.

E. None of the above

12. Find $\arcsin(\cos(\frac{3\pi}{4})) = \sin^{-1}(\cos(\frac{3\pi}{4}))$.

A. $\frac{\pi}{4}$

B. $-\frac{3\pi}{4}$

C. $-\frac{\pi}{4}$

D. $\frac{3\pi}{4}$

E. None of the above

13. Suppose that f is a continuous function on the interval $[0,5]$ and we know that

$$f(0) = 1,\ f(1) = -1,\ f(2) = 1,\ f(3) = -1,\ f(4) = 1,\ \text{and } f(5) = -1.$$

Which of the following statements are true for any such f?

A. The equation $f(x) = 1$ has exactly three solutions in the interval $[0,5]$.

B. There are at most five solutions of the equation $f(x) = 0$ in the interval $[0,5]$.

C. There are exactly five solutions of the equation $f(x) = 0$ in the interval $[0,5]$.

D. There are at least five solutions of the equation $f(x) = 0$ in the interval $[0,5]$.

E. The equation $f(x) = -1$ has at most three solutions in the interval $[0,5]$.

14. If $f(2) = 4$ and the derivative of f at $x = 2$ is 7, find the equation of the tangent line to the curve $y = f(x)$ at $x = 2$.

A. $y = 4 - 7(x - 2)$

B. $y = 7 + 4(x - 2)$

C. $y = 4 + 7(x - 2)$

D. $y = 2 - 7(x - 4)$

E. $y = 2 + 7(x - 4)$

Free Response Questions
Show all of your work

15. For each limit below, evaluate the limit or state that the limit does not exist. Guessing the limit based on a table of values will not receive credit. Show all of your work. An answer with no work will receive no credit.

(a) $\lim_{x \to 0} \frac{\sin^2(x)}{1 - \cos(x)}$

Solution:

$$\begin{aligned}
\lim_{x \to 0} \frac{\sin^2(x)}{1 - \cos(x)} &= \lim_{x \to 0} \frac{1 - \cos^2 x}{1 - \cos x} \\
&= \lim_{x \to 0} \frac{(1 - \cos x)(1 + \cos x)}{1 - \cos x} \\
&= \lim_{x \to 0} (1 + \cos x) \\
&= 2
\end{aligned}$$

(b) $\lim_{x \to 1} \frac{(2x + 1)(2x - 2)}{(x - 8)(x - 1)}$

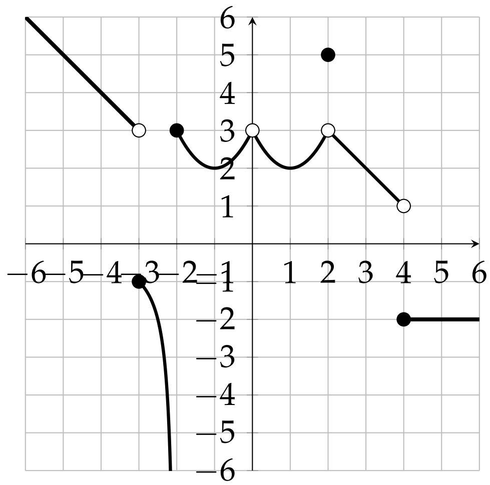
Solution:

$$\begin{aligned}
\lim_{x \to 1} \frac{(2x + 1)(2x - 2)}{(x - 8)(x - 1)} &= \lim_{x \to 1} \frac{(2x + 1)(2)(x - 1)}{(x - 8)(x - 1)} \\
&= \lim_{x \to 1} \frac{2(2x + 1)}{x - 8} \\
&= -\frac{6}{7}
\end{aligned}$$

(c) $\lim_{x \to 2^+} \frac{1}{x^2 + x - 6}$

Solution: The denominator goes to 0 at $x = 2$. Coming in from values greater than 2, the function tends to $+\infty$, or you could say that the limit does not exist as a real number.

$$\lim_{x \to 2^+} \frac{1}{x^2 + x - 6} = +\infty$$



16. The graph of $f(x)$ is shown above. Find the following limits if they exist or state that they do not exist.

(a) $\lim_{x \to -3^-} f(x)$

Solution: $\lim_{x \to -3^-} f(x) = 3$

(b) $\lim_{x \to -2^+} f(x)$

Solution: $\lim_{x \to -2^+} f(x) = 3$

(c) $\lim_{x \to 2} f(x)$

Solution: $\lim_{x \to 2} f(x) = 3$

(d) $\lim_{x \to 4} f(x)$

Solution: $\lim_{x \to 4} f(x)$ does not exist.

(e) What are the x-values at which $f(x)$ is not continuous on $[-6, 6]$.

Solution: $f(x)$ is not continuous at $x = -3$, $x - 2$, $x = 0$, $x = 2$, and at $x = 4$.

17. Let $f(x) = x^2 + 2x + 3$.

(a) Using the definition of the derivative, set up the limit that gives the derivative of f at $x = 2$.

Solution:

$$f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2} \frac{(x^2 + 2x + 3) - 11}{x - 2}.$$

Or

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{((2+h)^2 + 2(2+h) + 3) - 11}{h}.$$

(b) Compute the limit from (a). Show your work. An answer without supporting work will receive no credit.

Solution: First limit:

$$\begin{aligned} f'(2) &= \lim_{x \to 2} \frac{(x^2 + 2x + 3) - 11}{x - 2} = \lim_{x \to 2} \frac{x^2 + 2x - 8}{x - 2} \\ &= \lim_{x \to 2} \frac{(x+4)(x-2)}{x - 2} \\ &= \lim_{x \to 2} (x + 4) = 6 \end{aligned}$$

Second limit

$$\begin{aligned} f'(2) &= \lim_{h \to 0} \frac{((2+h)^2 + 2(2+h) + 3) - 11}{h} \\ &= \lim_{h \to 0} \frac{4 + 4h + h^2 + 4 + 2h + 3 - 11}{h} \\ &= \lim_{h \to 0} \frac{6h + h^2}{h} \\ &= \lim_{h \to 0} (6 + h) = 6 \end{aligned}$$

(c) Find the equation of the tangent line to $f(x)$ at $x = 2$.

Solution: The slope of the tangent line is $m = f'(2) = 6$ and the tangent line passes through the point $(2, f(2)) = (2, 11)$, so the equation of the tangent line is $y = 11 + 6(x - 2)$.