MA 113 Calculus I — Fall 2014
Exam 2 — Tuesday, 21 October 2014

Name: ______________________________

Section: ____________________________

Last 4 digits of student ID #: ________

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

D, A, A, E, E, A, B, C, B, A

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. Suppose $f(x) = x^{10} + 4x^5 + 2x^2 + 1$. Find $f''(1)$.

   (A) 91
   (B) 110
   (C) 134
   (D) 174
   (E) 230

2. Suppose $y = 3x + 2$ is the equation of the line tangent to a function f at the point $(1, f(1))$. Find $f(1)$ and $f'(1)$.

   (A) $f(1) = 5$ and $f'(1) = 3$
   (B) $f(1) = 3$ and $f'(1) = 5$
   (C) $f(1) = 3$ and $f'(1) = 2$
   (D) $f(1) = 2$ and $f'(1) = 3$
   (E) $f(1) = 5$ and $f'(1) = 2$

3. Suppose $f(x) = \tan^2(x)$. Find $f'(\frac{\pi}{4})$.

   (A) 4
   (B) 2
   (C) 1
   (D) 0
   (E) -2

Record the correct answer to the following problems on the front page of this exam.

4. Suppose $f(x) = e^{2x}\cos(3x)$. Find $f'(\frac{\pi}{2})$.

   (A) $-e^{\pi}$

   (B) $2e^{\pi}$

   (C) $-2e^{\pi}$

   (D) $-3e^{\pi}$

   (E) $3e^{\pi}$

5. Suppose that the function $y(x)$ satisfies the equation $xy^2 + 2y = 2x$. Find $\dfrac{dy}{dx}$ at the point $(2,1)$.

   (A) $\frac{1}{2}$

   (B) $\frac{1}{3}$

   (C) $\frac{1}{4}$

   (D) $\frac{1}{5}$

   (E) $\frac{1}{6}$

Record the correct answer to the following problems on the front page of this exam.

6. Suppose $f = \sqrt{2 + \sqrt{x}}$. Find $f'(4)$.

   (A) $\frac{1}{16}$
   (B) $\frac{1}{4}$
   (C) $\frac{1}{2}$
   (D) 1
   (E) 2

7. Suppose $f(x) = \dfrac{1 + x^2}{g(x) + x}$, and $g(0) = 2$ and $g'(0) = 3$. Find $f'(0)$.

   (A) -2
   (B) -1
   (C) 0
   (D) 1
   (E) 2

Record the correct answer to the following problems on the front page of this exam.

8. Let g be the inverse function of the function f. Suppose $f(0) = 2$, $f'(0) = 3$, $f(2) = 5$ and $f'(2) = 4$. Find $g(2)$ and $g'(2)$.

   (A) $g(2) = 0$ and $g'(2) = 4$

   (B) $g(2) = 0$ and $g'(2) = 1/4$

   (C) $g(2) = 0$ and $g'(2) = 1/3$

   (D) $g(2) = 5$ and $g'(2) = 1/4$

   (E) $g(2) = 5$ and $g'(2) = 1/3$

9. Suppose that $f(x) = e^x + 2e^{-x}$. Find $f^{(401)}(x)$.

   (A) $e^x + 2e^{-x}$

   (B) $e^x - 2e^{-x}$

   (C) $e^x + e^{-x}$

   (D) $e^x - e^{-x}$

   (E) None of the above

10. Suppose the radius of a sphere is $2t + 3$ meters (t is time and measured in seconds). Find the rate of change of the volume of the sphere measured in m^3/sec when $t = 3$. (The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.)

    (A) 648π

    (B) 728π

    (C) 512π

    (D) 484π

    (E) None of the above.

Free Response Questions: Show your work!

11. (a) Let $g(x) = \sec(e^{2x})$. Find $g'(x)$.

$g'(x) = \sec(e^{2x})\tan(e^{2x}) \cdot \frac{d(e^{2x})}{dx} = \sec(e^{2x})\tan(e^{2x})2e^{2x}$

(b) Find the coordinates of all points where the curve given by the equation

$$25x^2 + 16y^2 + 200x - 160y + 400 = 0$$

has horizontal tangent lines.

$50x + 32y\frac{dy}{dx} + 200 - 160\frac{dy}{dx} = 0$
$50x + 200 + (32y - 160)\frac{dy}{dx} = 0$
$\frac{dy}{dx} = \frac{-(50x+200)}{32y-160}$
If the tangent line is horizontal, then $\frac{dy}{dx} = 0$. Thus we must have $x = -4$. When $x = -4$, we have
$25(-4)^2 + 16y^2 + 200(-4) - 160y + 400 = 0$,
$16y^2 - 160y = 0$,
$16y(y - 10) = 0$.
Thus $y = 0, 10$. The points are $(-4, 0)$ and $(-4, 10)$.

Free Response Questions: Show your work!

12. For parts (a) and (b) below, assume that f and g are two functions such that $f(0) = 1$, $f'(0) = 0$, $g(0) = 1$ and $g'(0) = 2$.

    (a) Find $h'(0)$ where $h(x) = 3f(x)g(x) + [g(x)]^2$.

    $$h'(x) = 3(f(x)g'(x) + f'(x)g(x)) + 2g(x)g'(x)$$

    $$\begin{aligned} h'(0) &= 3(f(0)g'(0) + f'(0)g(0)) + 2g(0)g'(0) \\ &= 3(1 \cdot 2 + 0 \cdot 1) + 2 \cdot 1 \cdot 2 = 10 \end{aligned}$$

    (b) Find $l'(0)$ where $l(x) = \ln\left(\frac{f(x)}{g(x)}\right)$.

    $$\begin{aligned} l'(x) &= \frac{1}{\frac{f(x)}{g(x)}} \frac{d\left(\frac{f(x)}{g(x)}\right)}{dx} \\ &= \frac{g(x)}{f(x)}\left(\frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}\right) \end{aligned}$$

    $$\begin{aligned} l'(0) &= \frac{g(0)}{f(0)}\left(\frac{g(0)f'(0) - f(0)g'(0)}{g(0)^2}\right) \\ &= \frac{1}{1}\left(\frac{1 \cdot 0 - 1 \cdot 2}{1^2}\right) = -2 \end{aligned}$$

Free Response Questions: Show your work!

13. (a) Let $f(x) = \sqrt{x+1}$. Use the definition of the derivative as a limit to compute $f'(0)$. If you use any other method, you will not receive any points.

$f'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{\sqrt{1+h}-\sqrt{1}}{h}$
$= \lim_{h\to 0} \frac{\sqrt{1+h}-1}{h} \cdot \frac{\sqrt{1+h}+1}{\sqrt{1+h}+1} = \lim_{h\to 0} \frac{h}{h(\sqrt{1+h}+1)}$
$= \lim_{h\to 0} \frac{1}{\sqrt{1+h}+1} = \frac{1}{2}$

(b) Find constants a and b such that the function

$$f(x) = \begin{cases} ax+b, & x \le 1 \\ -(x-2)^2+4, & x > 1 \end{cases}$$

is differentiable at $x = 1$.

If f is differentiable at $x = 1$, then f is continuous at $x = 1$. Thus $f(1) = 3$. We need $\lim_{h\to 1^-} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 1^+} \frac{f(1+h)-f(1)}{h}$. This requires $a = -2(1-2)$ and so $a = 2$. We now have $3 = f(1) = a \cdot 1 + b = 2 + b$. Thus $b = 1$.

Free Response Questions: Show your work!

14. A ladder of length 290 cm is leaning against a wall. The ladder begins to slide away from the wall. The bottom of the ladder moves away from the wall at a constant rate of 60 cm/sec. Find the rate of change of the height of the top of the ladder when the height of the top of the ladder is 200 cm.

Let x denote the distance from the bottom of the ladder to the wall. Let h denote the height of the top of the ladder. Then $h^2 + x^2 = 290^2$.
$2h\frac{dh}{dt} + 2x\frac{dx}{dt} = 0$.
We are given that $\frac{dx}{dt} = 60$. When $h = 200$, we have $x = \sqrt{290^2 - 200^2} = 210$. Substituting gives $2 \cdot 200\frac{dh}{dt} + 2 \cdot 210 \cdot 60 = 0$. Thus $\frac{dh}{dt} = -210(60)/200 = -63$ cm/sec.

Free Response Questions: Show your work!

15. The position (measured in meters) of an object moving in a straight line is given by the equation
$$P(t) = \frac{1}{3}t^3 - \frac{3}{2}t^2 - 2t + 10,\ t \geq 0.$$

(a) Find the time when the velocity of the object is 2 m/s.

$v(t) = P'(t) = t^2 - 3t - 2$. We require that $t^2 - 3t - 2 = 2$, so $t^2 - 3t - 4 = 0$. This gives $(t+1)(t-4) = 0$. Thus $t = 4$ because $t \geq 0$.

(b) For what values of t is the acceleration of the object positive?

$a(t) = P''(t) = 2t - 3$. We require that $2t - 3 > 0$. Thus $t > 3/2$.