MA 113 Calculus I — Fall 2015
Exam 2 — Tuesday, 20 October 2015

**Name:** ______________________

**Section:** ______________________

**Last 4 digits of student ID #:** ________

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-phones during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

**On the multiple choice problems:**

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box.*

**On the free response problems:**

- Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

**Multiple Choice Answers**

| Question | | | | | |
|---|---|---|---|---|---|
| 1 | A | B | C | D | E |
| 2 | A | B | C | D | E |
| 3 | A | B | C | D | E |
| 4 | A | B | C | D | E |
| 5 | A | B | C | D | E |
| 6 | A | B | C | D | E |
| 7 | A | B | C | D | E |
| 8 | A | B | C | D | E |
| 9 | A | B | C | D | E |
| 10 | A | B | C | D | E |

[D,A,C,E,B,E,D,C,A,B]

**Exam Scores**

| Question | Score | Total |
|---|---|---|
| MC | | 50 |
| 11 | | 10 |
| 12 | | 10 |
| 13 | | 10 |
| 14 | | 10 |
| 15 | | 10 |
| Total | | 100 |

1. Suppose $f(x) = (x^3 + 1)e^{-x}$. Find $f'(1)$.

   (A) $5e^{-1}$

   (B) $-5e^{-1}$

   (C) $-e^{-1}$

   (D) $e^{-1}$

   (E) $3e^{-1}$

2. Suppose $f(x) = x^3 - 2x + 5x^{-1}$. Find an equation of the tangent line to the graph of $y = f(x)$ at the point $(1, 4)$.

   (A) $y = -4x + 8$

   (B) $y = 4x$

   (C) $y = x + 3$

   (D) $y = 4$

   (E) $y = 3x + 1$

**Record the correct answer to the following problems on the front page of this exam.**

3. Suppose $f(x) = x\cos x + \sin x$. Find $f''(x)$.

   (A) $2\cos x + x\sin x$

   (B) $-4\cos x - x\sin x$

   (C) $-3\sin x - x\cos x$

   (D) $-3\sin x + x\cos x$

   (E) $-2\cos x + 3x\sin x$

4. Suppose $y = \ln(9t^2 + 2t + 5)$. Find $y'(1)$.

   (A) $\dfrac{4}{5}$

   (B) $\dfrac{1}{2}$

   (C) $4\ln 2$

   (D) $\dfrac{9}{16}$

   (E) $\dfrac{5}{4}$

**Record the correct answer to the following problems on the front page of this exam.**

5. Suppose $f(x) = \sin^2(x^3 + 1)$. Find $f'(x)$.

   (A) $x^3 \sin(x^3 + 1)$

   (B) $6x^2 \sin(x^3 + 1) \cos(x^3 + 1)$

   (C) $2 \sin(x^3 + 1) \cos(x^3 + 1)$

   (D) $3x^5 \sin(x^3 + 1)$

   (E) $3x^2 \sin(x^3 + 1) \cos(3x^2)$.

6. Suppose that $y = f(x)$ and $y^3x^4 - 10x + y = -3$. Find $\dfrac{dy}{dx}$ at $P = (2, 1)$.

   (A) $-\dfrac{11}{24}$

   (B) $\dfrac{15}{41}$

   (C) $-\dfrac{16}{25}$

   (D) $-\dfrac{21}{48}$

   (E) $-\dfrac{22}{49}$

**Record the correct answer to the following problems on the front page of this exam.**

7. Find $f'(x)$, where $f(x) = \sqrt{x^2 + \cos^2 x}$.

   (A) $\frac{1}{2}(x^2 + \cos^2 x)^{-1/2}$

   (B) $x(x^2 + \cos^2 x)^{-1/2}$

   (C) $(x^2 + \cos^2 x)^{-1/2}(x + \sin x \cos x)$

   (D) $(x^2 + \cos^2 x)^{-1/2}(x - \sin x \cos x)$

   (E) $(x^2 + \cos^2 x)^{1/2}(x + \sin x \cos x)$

8. Find $f'(x)$, if $f(x) = \dfrac{e^{3x}}{x+1}$.

   (A) $\dfrac{xe^{3x}}{(x+1)^2}$

   (B) $\dfrac{3xe^{3x}}{(x+1)^2}$

   (C) $\dfrac{(3x+2)e^{3x}}{(x+1)^2}$

   (D) $\dfrac{(x+3)e^{3x}}{(x+1)^2}$

   (E) $\dfrac{(3x+1)e^{3x}}{(x+1)^2}$

**Record the correct answer to the following problems on the front page of this exam.**

9. Find $g'(e)$, where $g(x)$ is the inverse of $f(x) = x^3 e^{x^2}$. Hint: $f(1) = e$.

   (A) $\frac{1}{5e}$

   (B) $\frac{5}{e}$

   (C) $-\frac{5}{e}$

   (D) $-\frac{1}{5e}$

   (E) $\frac{1}{4e}$

10. Recall that $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$ and $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$. Find $\frac{dy}{dx}$, where $y = \arctan\left(\frac{x}{4}\right)$.

    (A) $\frac{1}{4(1+x^2)}$

    (B) $\frac{4}{16+x^2}$

    (C) $\frac{4}{\sqrt{16-x^2}}$

    (D) $\frac{4}{1+x^2}$

    (E) $\frac{4}{\sqrt{1-x^2}}$

11. A road perpendicular to a highway leads to a farmhouse located 16 km from the highway. An automobile traveling on the highway passes through this intersection at a speed of 90 km/h.

    How fast is the distance between the automobile and the farmhouse increasing when the automobile is 12 km past the intersection of the highway and the road? (Draw a picture and label the picture to represent the situation in this problem.)

    Let $D$ denote the distance between the automobile and the farmhouse and let $x$ denote the distance of the automobile past the intersection. Then $D^2 = x^2 + 16^2$, and thus $2DD'(t) = 2xx'(t)$. This gives $D'(t) = \dfrac{xx'(t)}{D} = \dfrac{xx'(t)}{\sqrt{x^2 + 16^2}}$. We are given that $x'(t) = 90$. When $x = 12$, we have $D = \sqrt{400} = 20$. Therefore, when $x = 12$, we have $D'(t) = \frac{12 \cdot 90}{20} = 54$ km/h.

## Free Response Questions: Show your work!

12. (a) Assume that $f(x)$ is a differentiable function. The equation of the tangent line at $x = 7$ to the graph of $y = f(x)$ is $y = 3x - 10$. Find $f(7)$ and $f'(7)$.

    The equation of the tangent line is given by $y - f(7) = f'(7)(x - 7)$. Thus $y = f'(7)x + (f(7) - 7f'(7))$. The slope of the tangent line, which is 3, is given by $f'(7)$. Thus $-10 = f(7) - 7f'(7) = f(7) - 21$. This gives $f(7) = 11$ and $f'(7) = 3$.

    (b) Find the coordinates of the points on the graph of $y = x^3 + 6x^2 + 5x + 2$ where the tangent line is parallel to the line $y = -4x + 5$.

    We must solve $y' = 3x^2 + 12x + 5 = -4$ because the slope of the tangent line must equal $-4$. This gives $3x^2 + 12x + 9 = 0$, $x^2 + 4x + 3 = 0$, $(x + 1)(x + 3) = 0$. Then $x = -1, -3$. This leads to the points $(-1, 2)$ and $(-3, 14)$.

## Free Response Questions: Show your work!

13. An object is dropped from a height of 640 meters. As usual, we ignore all air resistance in this problem. Use Galileo's formula $s(t) = s_0 + v_0 t - \frac{1}{2} g t^2$, where $g = 9.8\, m/s^2$, to answer the following questions. Be sure to use correct units.

    (a) When does the object hit the ground?

    $s_0 = 640$ and $v_0 = 0$. Then we must solve the equation $0 = s(t) = 640 - 4.9t^2$. This gives $t^2 = \frac{640}{4.9} = \frac{6400}{49}$. Then $t = \frac{80}{7}$ seconds.

    (b) What is the object's velocity when it hits the ground?

    $v(t) = v_0 - gt = -gt$. The velocity when the object hits the ground is $v(80/7) = -9.8 \cdot 80/7 = -112$ m/s.

**Free Response Questions: Show your work!**

14. (a) State the definition of the derivative of a function $f(x)$.

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$.

(b) Compute the derivative of $f(x) = \frac{1}{x^2}$ using the definition of a derivative as a limit. (You will not receive any credit if you use a different method.)

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \lim_{h\to 0} \frac{x^2 - (x+h)^2}{hx^2(x+h)^2} \\ &= \lim_{h\to 0} \frac{-2xh - h^2}{hx^2(x+h)^2} = \lim_{h\to 0} \frac{-2x - h}{x^2(x+h)^2} \\ &= \frac{-2x}{x^4} = -2x^{-3}. \end{aligned}$$

**Free Response Questions: Show your work!**

15. An expanding sphere has radius $r = t^2 + t$ cm at time $t$ in seconds, $t > 0$. In this problem you may use the formula $V = \frac{4}{3}\pi r^3$ for the volume of a sphere of radius $r$. Be sure to use correct units.

(a) Find the rate of change of the volume of the sphere with respect to time when the radius is 2 cm.

$\frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$. We have $\frac{dr}{dt} = 2t + 1$. When $r = 2$, we have $t^2 + t = 2$, $t^2 + t - 2 = 0$, $(t+2)(t-1) = 0$, $t = 1$ because $t > 0$. Then $\frac{dV}{dt}|_{t=1,\, r=2} = 4\pi \cdot 2^2 \cdot 3 = 48\pi$ cm$^3$/s.

(b) Find the rate of change of the volume of the sphere with respect to time when $t = 3$.

When $t = 3$ we have $r = 12$. Then $\frac{dV}{dt}|_{t=3,\, r=12} = 4\pi \cdot 12^2 \cdot 7 = 4032\pi$ cm$^3$/s.