MA 113 Calculus I                    Fall 2018

Exam 2                    Tuesday, 16 October 2018

**Name:** _____

**Section:** _____

**Last 4 digits of student ID #:** _____

This exam has 12 multiple choice questions (five points each) and 4 free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-buds during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

**On the multiple choice problems:**

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.

- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

**On the free response problems:**

- Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,

- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

**Multiple Choice Answers**

| Question | | | | | |
|----------|---|---|---|---|---|
| 1  | A | B | C | D | E |
| 2  | A | B | C | D | E |
| 3  | A | B | C | D | E |
| 4  | A | B | C | D | E |
| 5  | A | B | C | D | E |
| 6  | A | B | C | D | E |
| 7  | A | B | C | D | E |
| 8  | A | B | C | D | E |
| 9  | A | B | C | D | E |
| 10 | A | B | C | D | E |
| 11 | A | B | C | D | E |
| 12 | A | B | C | D | E |

**Exam Scores**

| Question | Score | Total |
|----------|-------|-------|
| MC    |  | 60  |
| 13    |  | 10  |
| 14    |  | 10  |
| 15    |  | 10  |
| 16    |  | 10  |
| Total |  | 100 |

1. Suppose the tangent line to $f(x)$ has equation $y = 7x - 1$. Find $f(3)$ and $f'(3)$.

   (A) $f(3) = 20$ and $f'(3) = 7$

   (B) $f(3) = -1$ and $f'(3) = 7$

   (C) $f(3) = 7$ and $f'(3) = -1$

   (D) $f(3) = 7$ and $f'(3) = 20$

   (E) None of the above

2. The limit below represents a derivative $f'(a)$. Find $f(x)$ and $a$.

$$\lim_{h \to 0} \frac{\cos(\frac{\pi}{2} + h)}{h}$$

   (A) $f(x) = \sin(x)$ and $a = \pi/2$

   (B) $f(x) = \cos(\pi/2 + h)$ and $a = 0$

   (C) $f(x) = \cos(x)$ and $a = \pi/2$

   (D) $f(x) = \cos(x + h)$ and $a = \pi/2$

   (E) None of the above

3. Determine coefficients $a$ and $b$ such that $p(x) = x^2 + ax + b$ satisfies $p(1) = 2$ and $p'(1) = 2$.

   (A) $a = 1$ and $b = 1$

   (B) $a = 1$ and $b = 2$

   (C) $a = 1$ and $b = 0$

   (D) $a = 0$ and $b = 1$

   (E) None of the above

4. Suppose that $f(4) = 3$, $f'(4) = -8$, $g(4) = 4$, and $g'(4) = -3$. Find $G'(4)$ where $G(x) = x \cdot g(x) \cdot f(x)$.

   (A) -29

   (B) -152

   (C) -164

   (D) 128

   (E) None of the above

5. Let $f(x)$ be the function whose graph is shown below. For which $x$-values in the interval $(0, 7)$ is $f(x)$ not differentiable?
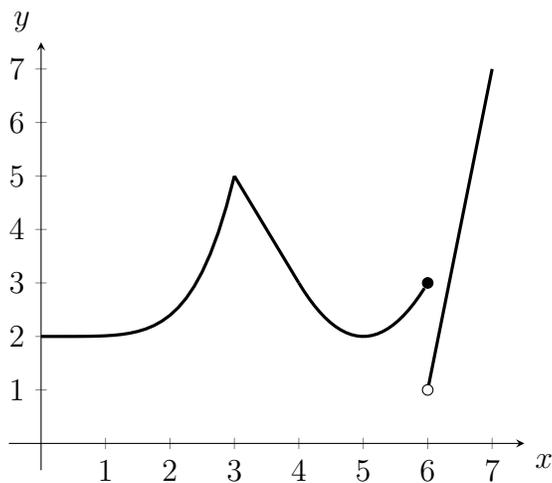


   (A) 3 and 6

   (B) 6

   (C) 3 and 5 and 6

   (D) 5

   (E) None of the above.

6. Evaluate the limit $\lim\limits_{x \to 0} \dfrac{\sin(3x)}{7x}$.

   (A) 1/7

   (B) 3

   (C) 7/3

   (D) 1

   (E) None of the above

7. Find the derivative of $f(x) = e^{\cos(x^2)}$

   (A) $-2x \sin(x^2) e^{\cos(x^2)}$

   (B) $e^{\cos(x^2)}$

   (C) $2x \sin(x^2) e^{\cos(x^2)}$

   (D) $\sin(x^2) e^{\cos(x^2)}$

   (E) None of the above.

8. Find $\dfrac{dy}{dx}$ when $x^2 y + 2xy^2 = x + y$.

   (A) $(-2xy - 2y^2 - 1)/(x^2 - 4xy - 1)$

   (B) $(-2xy - 2y^2 + 1)/(x^2 + 4xy - 1)$

   (C) $2y + 2y^2 - 1 - y$

   (D) $(-2xy - 2y + 1)/(x^2 + 4xy - 1)$

   (E) None of the above

9. Find the derivative of $f(x) = \dfrac{\ln(6x)}{6x}$.

   (A) $\dfrac{\ln(6)}{6}$

   (B) $\dfrac{x - \log(6x)}{36x^2}$

   (C) $\dfrac{1 - \log(6x)}{6x^2}$

   (D) 0

   (E) None of the above

10. Find the rate of change of the volume of a cube with respect to the length of its side $s$ when $s = 27$.

    (A) $19,683$ cubic meters per meter

    (B) $729$ cubic meters per meter

    (C) $2187$ cubic meters per meter

    (D) $27$ cubic meters per meter

    (E) None of the above

11. A person of height 2 meters walks away from a 5-meter tall lamppost at a speed of $1/2$ meters per second. Find the rate at which the person's shadow is increasing in length.

    (A) $2/3$ meters per second

    (B) $1$ meters per second

    (C) $1/3$ meters per second

    (D) $3$ meters per second

    (E) None of the above.

12. Find the third derivative of $f(x) = \sin(3x + 1)$.

    (A) $27 \cos(3x + 1)$

    (B) $-\cos(3x + 1)$

    (C) $-27 \cos(3x + 1)$

    (D) $-27 \cos(x)$

    (E) None of the above

13. (a) Suppose a rectangular bathtub has base area 30 square meters and the water is filled to height $h$ meters. Find the volume of water as a function of $h$. Include units!

(b) Suppose the height of the water is changing at a rate of 0.7 meters per second. At what rate is water being poured into the bathtub? Include units!

14. (a) Suppose that $x^4 + y^4 = 2$. Find the slope of the tangent line to the curve defined by this equation at the point $(1, -1)$.

(b) Use implicit differentiation to find the derivative of the inverse cosine function, which is denoted as $\arccos(x)$ or $\cos^{-1}(x)$. NOTE: Writing only the formula for the derivative of $\arccos(x)$ will receive no credit, you must use implicit differentiation on the equation $\cos(\cos^{-1}(x)) = x$.

15. Let $f(x) = \cos(\pi \cdot e^x)$.

    (a) Find the derivative $f'(x)$.

    (b) Find the equation of the tangent line to $f(x)$ at the point where $x = 0$.

16. This problem concerns the definition of the derivative using limits.

    (a) State the formal definition of the derivative of a function $f(x)$ at the point $x = a$.
    *Hint*: Your definition should involve a limit.

    (b) **Using the formal definition of derivative and the limit laws**, find the derivative of the function $f(x) = 2x^2 + 1$. An answer that is unsupported or uses differentiation rules will receive **no credit**.