# *Exam 2*

Form A

KEY

Multiple Choice Questions

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | A | B | C | D | E | **9** | A | B | C | D | E |
| **2** | A | B | C | D | E | **10** | A | B | C | D | E |
| **3** | A | B | C | D | E | **11** | A | B | C | D | E |
| **4** | A | B | C | D | E | **12** | A | B | C | D | E |
| **5** | A | B | C | D | E | **13** | A | B | C | D | E |
| **6** | A | B | C | D | E | **14** | A | B | C | D | E |
| **7** | A | B | C | D | E | **15** | A | B | C | D | E |
| **8** | A | B | C | D | E | **16** | A | B | C | D | E |

# SCORE

| Multiple Choice | 17 | 18 | 19 | 20 | Total Score |
|---|---|---|---|---|---|
| 64 | 9 | 9 | 9 | 9 | 100 |
| | | | | | |

## Multiple Choice Questions

1. Find $f(3)$ and $f'(3)$, assuming that the tangent line to $y = f(x)$ at $x = 3$ has equation $y = 4x - 3$.

   A. $f(3) = -3$, $f'(3) = 4$

   B. $f(3) = 3$, $f'(3) = 4$

   C. $f(3) = 4$, $f'(3) = -3$

   **D.** $f(3) = 9$, $f'(3) = 4$

   E. $f(3) = 15$, $f'(3) = 4$

2. Determine coefficients $a$ and $b$ such that $p(x) = x^2 + ax + b$ satisfies $p(2) = 9$ and $p'(2) = 8$.

   A. $a = 0$, $b = 5$

   B. $a = 1/4$, $b = 4$

   **C.** $a = 4$, $b = -3$

   D. $a = 4$, $b = 3$

   E. $a = 12$, $b = -19$

3. Find a formula for $\frac{dy}{dx}$ in terms of $x$ and $y$, where $x^3y + 3xy^3 = 26$.

   A. $\frac{dy}{dx} = -\frac{x^3 + 9xy^2}{3x^2y + 3y^3}$

   B. $\frac{dy}{dx} = \frac{3x^2y + 3y^3}{x^3 + 9xy^2}$

   C. $\frac{dy}{dx} = \frac{x^3 + 9xy^2}{3x^2y + 3y^3}$

   D. $\frac{dy}{dx} = \frac{26}{3x^2 + 9y^2}$

   **E.** $\frac{dy}{dx} = -\frac{3x^2y + 3y^3}{x^3 + 9xy^2}$

4. Suppose $f(2) = 3$ and $f'(2) = 5$ and let $g(x) = f(x)/x$. Find $g'(2)$.

   A. $g'(2) = -13/2$

   B. $g'(2) = -7/2$

   C. $g'(2) = -7/4$

   **D.** $g'(2) = 7/4$

   E. $g'(2) = 7/2$

5. Find the slope of the tangent line to the graph of $f(x) = x^2e^{2x}$ at $x = 2$

   A. $4e^2$

   B. $8e^2$

   C. $4e^4$

   D. $8e^4$

   **E.** $12e^4$

6. Find the derivative of $g(x) = x^3 \ln(x^2)$.

   **A.** $g'(x) = 3x^2 \ln(x^2) + 2x^2$

   B. $g'(x) = 3x^2 \ln(2x)$

   C. $g'(x) = 6x$

   D. $g'(x) = 3x^2 \ln(x^2) + 2x^4$

   E. $g'(x) = 3x^2 \cdot \dfrac{1}{2x}$

7. Let $f(x) = x^4 + 3x - 1$ and let $g$ be the inverse of $f$. Find $g(3)$ and $g'(3)$.

A. $g(3) = -1, g'(3) = 3$

**B.** $g(3) = 1, g'(3) = 1/7$

C. $g(3) = 1, g'(3) = 7$

D. $g(3) = 89, g'(3) = 1/111$

E. $g(3) = 89, g'(3) = 111$

8. Find the equation of the tangent line to $3x^2 + 5y^3 = 8$ at $(4, -2)$.

A. $y = -\frac{3}{5}x + \frac{2}{5}$

**B.** $y = -\frac{2}{5}x - \frac{2}{5}$

C. $y = -\frac{1}{5}x - \frac{6}{5}$

D. $y = \frac{1}{5}x - \frac{14}{5}$

E. $y = \frac{2}{5}x - \frac{18}{5}$

9. Let $h(x) = \dfrac{\cos(x)}{2 + x + x^2}$. What is $h'(0)$?

   A. $-1/2$

   **B. $-1/4$**

   C. $0$

   D. $1/4$

   E. $1/2$

10. A watermelon is dropped off a tall building so that its height in meters at time $t$ in seconds is $h(t) = -4.9t^2 + 200$. Find the velocity when it hits the ground. Give your answer correctly rounded to one decimal place.

    A. $-62.4$ meters per second.

    **B. $-62.6$ meters per second.**

    C. $-62.8$ meters per second.

    D. $-63$ meters per second.

    E. $-63.2$ meters per second.

11. Find the points $c$ (if any) such that $f'(c)$ does not exist where $f(x) = |2x - 4|$.

A. $c = -2$

B. $c = -\frac{1}{2}$

C. $c = \frac{1}{2}$

**D.** $c = 2$

E. There are no such points for this function.

12. Let $f$ and $g$ be two function and let $h(x) = f(g(x))$. If $g(2) = 3$, $g'(2) = 5$, $f(2) = 7$, $f'(2) = 1$, $f(3) = -1$, and $f'(3) = -2$, what is $h'(2)$?

**A.** $-10$

B. $-1$

C. 3

D. 5

E. 38

13. Find all the values of $x$ where $f''(x) = 0$ when $f(x) = 3xe^x$.

**A.** $-2$

B. $-1/2$

C. $0$

D. $1/2$

E. $2$

14. Find the derivative of $g(x) = \tan(\sin(3x))$.

A. $g'(x) = -3\cos(3x)\sec^2(\sin(3x))$

B. $g'(x) = \cos(3x)\sec^2(\sin(3x))$

C. $g'(x) = \sec^2(3\cos(3x))$

**D.** $g'(x) = 3\cos(3x)\sec^2(\sin(3x))$

E. $g'(x) = 3\sec^2(\cos(3x))$

15. If $g(0) = 4$ and $g'(0) = 2$, then find the derivative of $f(x) = e^{xg(x)}$ when $x = 0$.

A. $f'(0) = 1/4$

B. $f'(0) = 1/2$

C. $f'(0) = 0$

D. $f'(0) = 2$

**E.** $f'(0) = 4$

16. Strontium-90 has a half-life of 28 days. A sample has a mass of 100 mg initially. Find the mass remaining after 50 days rounded to two decimal places.

A. 8.41 mg

B. 28.00 mg

**C.** 29.00 **mg**

D. 50.00 mg

E. 68.04 mg

Free Response Questions
**Show all of your work**

17. (a) Find all points on $y^2 + 2xy + 2x^2 = 8$ with $x = 2$.

**Solution:** Setting $x = 2$ we have the equation $y^2 + 4y + 8 = 8$ or $y(y+4) = 0$. The points with $x = 2$ are the points $(2, 0)$ and $(2, -4)$.

(b) Find $dy/dx$ for $y^2 + 2xy + 2x^2 = 8$

**Solution:**

$$\begin{aligned} y^2 + 2xy + 2x^2 &= 8 \\ 2y\frac{dy}{dx} + \left(2y + 2x\frac{dy}{dx}\right) + 4x &= 0 \\ \frac{dy}{dx}(2y + 2x) &= -4x - 2y \\ \frac{dy}{dx} &= -\frac{2x + y}{x + y} \end{aligned}$$

(c) Find the slope of the tangent line to $y^2 + 2xy + 2x^2 = 8$ at each point with $x = 2$.

**Solution:** Using the derivative from above we have:

$x = 2, y = 0$: $m = \left.\frac{dy}{dx}\right|_{(x,y)=(2,0)} = -2.$

$x = 2, y = -4$: $m = \left.\frac{dy}{dx}\right|_{(x,y)=(2,-4)} = 0$

18. Suppose that $A_0 = 1000$ dollars in principal is invested in an account earning 4% interest, compounded continuously. Recall that the value of the account after $t$ years is $A(t) = A_0 e^{rt}$.

(a) What is the value of the account after 5 years?

**Solution:** $A = 1000e^{0.04\times 5} = \$1221.40$

(b) What is the rate of change of the value $A(t)$ at $t = 5$?

**Solution:** The rate of change is the derivative at $t = 5$.

$$\begin{aligned} A(t) &= 1000e^{0.04t} \\ A'(t) &= 40e^{0.04t} \\ A'(5) &= 40e^{0.2} = 48.856 \text{ dollars per year} \end{aligned}$$

(c) What is the rate of change of the value $A(t)$ when $A(t) = 1500$?

**Solution:** First, find $t$ so that $A(t) = 1500$.

$$\begin{aligned} A(t) &= 1500 \\ 1000e^{0.04t} &= 1500 \\ t &= \frac{\ln 1.5}{0.04} = 10.137 \text{ years} \\ A'(t) &= 40e^{0.04t} \\ A'(10.137) &= 60 \text{ dollars per year} \end{aligned}$$

19. Find the derivatives of the following functions

(a) $f(x) = \ln(\sin(3x))$

**Solution:** $f'(x) = \dfrac{3\cos(3x)}{\sin(3x)}$

(b) $g(x) = \dfrac{4}{x^3} - \dfrac{3}{x^2} + \dfrac{2}{x} + 4$

**Solution:** $g'(x) = -\dfrac{12}{x^4} + \dfrac{6}{x^3} - \dfrac{2}{x^2}$

(c) $h(x) = 2\ln\left(\dfrac{x^2}{e^{3x}}\right)$

**Solution:** $h(x) = 2\left(\ln(x^2) - \ln(e^{3x})\right) = 4\ln x - 6x$ so $h'(x) = \dfrac{4}{x} - 6.$

20. Let $f(x) = \sqrt{3x+1}$.

(a) Find the instantaneous rate of change of $f(x)$ when $x = 5$.

**Solution:** $f'(x) = \frac{1}{2}(3x+1)^{-1/2}(3) = \frac{3}{2\sqrt{3x+1}}$ so $f'(5) = \frac{3}{8}$.

(b) Find the equation of the tangent line to $f(x)$ when $x = 5$.

**Solution:** We have from above that $f'(5) = \frac{3}{8}$, so the slope of the tangent line is $\frac{3}{8}$. Now, $f(5) = 4$ so the equation of the tangent line is

$$y = 4 + \frac{3}{8}(x-5).$$

(c) Find $f''(5)$.

**Solution:**

$$\begin{aligned} f(x) &= \sqrt{3x+1} \\ f'(x) &= \frac{3}{2}(3x+1)^{-1/2} \\ f''(x) &= -\frac{9}{4}(3x+1)^{-3/2} \\ f''(5) &= -\frac{9}{4} \times \frac{1}{64} \\ &= -\frac{9}{256} \end{aligned}$$