MA 113 — Calculus I Spring 2013
Exam 2 March 5, 2013

Name: ______________________________

Section: __________

Last 4 digits of student ID #: __________

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your final answers in the multiple choice answer box on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the multiple choice answer box.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer (unsupported answers may not receive credit).
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

Question					
1					E
2			C		
3	A				
4				D	
5			C		
6		B			
7	A				
8	A				
9		B			
10				D	

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Version 2013-S-2

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Version 2013-S-2b

THIS IS VERSION 2013-S-2 OF THE TEST.

(1) If $f(x) = 3e^{2x}$, then the fourth derivative of f at $x = 0$, $f^{(4)}(0)$, is

A) 6

B) 12

C) 16

D) 24

E) 48

(2) What is $\dfrac{d}{dx}\ln(\sin x))$?

A) $\tan x$

B) $\dfrac{\sin x}{x}$

C) $\cot x$

D) $\cos\left(\dfrac{1}{x}\right)$

E) None of the above.

(3) What does the limit $\lim_{x\to 2} \dfrac{x^{-2} - \frac{1}{4}}{x-2}$ represent?

A) $\left.\dfrac{d}{dx}\left(\dfrac{1}{x^2}\right)\right|_{x=2}$

B) 0

C) $\ln 2$

D) $-\dfrac{1}{2}$

E) The limit does not exist.

(4) Suppose that h is a differentiable function on $[-1, 2]$ satisfying $h(-1) = -1$, $h(0) = -2$, $h(1) = 1$ and $h(2) = 2$. Which of the following statements is NOT true?

A) There exists a point c in $[-1, 1]$ such that $h(c) = 0$.

B) There exists a point c in $[0, 1]$ such that $h(c) = 0$.

C) There exists a point c in $[0, 2]$ such that $h(c) = 0$.

D) The function h is one-to-one on the interval $[-1, 2]$.

E) The function h is continuous on the interval $[-1, 2]$.

(5) Consider the function f defined by

$$f(x) = \begin{cases} \arccos x & \text{if } x \leq 0 \\ ax + b & \text{if } x > 0 \end{cases},$$

where a and b are constants. Find a and b such that f is differentiable at $x = 0$.

A) $a = 1,\ b = \dfrac{\pi}{2}$

B) $a = 1,\ b = -2$

C) $a = -1,\ b = \dfrac{\pi}{2}$

D) $a = -1,\ b = 0$

E) $a = \dfrac{\pi}{2},\ b = 1$

(6) What is $g'(2)$ if $g(x) = \dfrac{2x^2 + 3}{f(x)}$, $f(2) = 1$, and $f'(2) = -3$?

A) -7

B) 41

C) -41

D) 7

E) None of the above.

(7) What is $h'(0)$ if $h(x) = (x - f(x))(2x+1)$, $f(0) = -2$, and $f'(0) = 0$?

A) 5

B) 1

C) 2

D) -3

E) None of the above.

(8) What is the slope of the line tangent to the curve $y = \tan x$ at $x = \dfrac{\pi}{4}$?

A) 2

B) $\sqrt{2}$

C) $\dfrac{1}{\sqrt{2}}$

D) $\dfrac{1}{2}$

E) None of the above.

(9) What is the derivative of $y = \sqrt{\sin(e^x))}$?

A) $\dfrac{e^x \cot(e^x)}{2}$

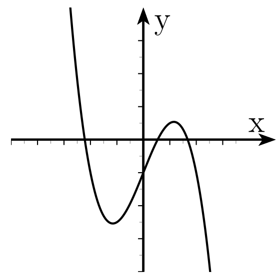
B) $\dfrac{e^x \cos(e^x)}{2\sqrt{\sin(e^x))}}$

C) $\dfrac{e^x \sin(e^x)\cos(e^x)}{2}$

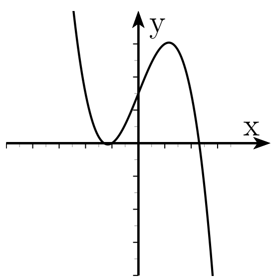
D) $\dfrac{\cos(e^x)}{2\sqrt{\sin(e^x))}}$

E) $\dfrac{e^x}{2\sqrt{\sin(e^x))}}$

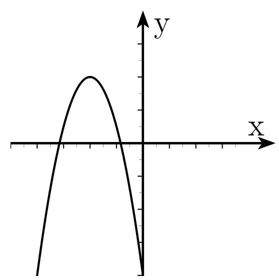
(10) The graph of a function $f(x)$ is shown on the right. Which of the graphs (A) - (E) could be the graph of $f'(x)$, the derivative of $f(x)$?



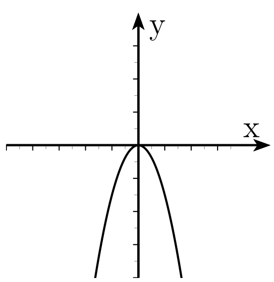
(A)



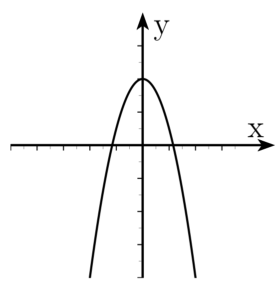
(B)



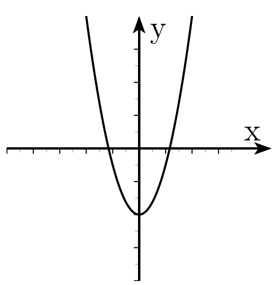
(C)



(D)



(E)



Free Response Questions: Show your work!

(11) Consider the equation

$$x^5 - x^2 + 2x - 3 = 0.$$

(a) (5 points) Show that the equation above has at least one real root. Explain your reasoning carefully.

Solution: Let $f(x) = x^5 - x^2 + 2x - 3$.

$f(0) = -3 < 0$

1 point: Find an endpoint where f is negative

$f(2) = 29 > 0$

1 point: Find an endpoint where f is positive

$f(x) = x^5 - x^2 + 2x - 3$ is a polynomial so it is continuous for all real numbers. By the Intermediate Value Theorem, there exists at least one value $c \in (0, 2)$ such that $f(c) = 0$. Alternatively, by the Existence of Zeros corollary, $f(x)$ has a zero between $(0, 2)$.

3 points: Invocation of IVT. Address continuity of f over the real numbers or over the interval $[a, b]$

(b) (5 points) Use the bisection method to find an interval of length less than or equal to 0.25 that contains a root of the equation above.

Solution:

$f(x) = x^5 - x^2 + 2x - 3$ has a zero in $(0, 2)$ by part (a). The midpoint of this interval is $x = 1$ and $f(1) < 0$

1 point: Identify midpoint of the interval from part (a) and determine sign of $f(x)$ at midpoint

$f(1) < 0$ and $f(2) > 0$, so the zero of $f(x)$ is in $(1, 2)$

2 points: Determine which subinterval contains the zero of f based on the previous step

$f(1.5) > 0$ and $f(1) < 0$, so the zero of $f(x)$ lies in $(1, 1.5)$. Finally, $f(1.25) > 0$ so the zero lies in $(1, 1.25)$.

2 points: Repeat process until the interval has length less than or equal to 0.25

Award **5 points** if a student provides an interval of suitable length and uses IVT.

(12) A particle is moving in a straight line. Its position after t seconds is given by $s(t) = \frac{2}{3}t^3 - 4t^2 + 6t$ meters for $t \geq 0$.

(a) (4 points) Find the time interval(s) when the particle is moving to the right.

Solution:

$v(t) = s'(t) = 2t^2 - 8t + 6$	**1 point: Correct derivative**
Particle moving to the right when $v(t) = 2(t-3)(t-1) > 0$	**1 point: Set** $v(t) > 0$
$0 < t < 1$ and $t > 3$	**2 points: Correct answer**

(b) (4 points) What is the total distance traveled by the particle over the time interval $[0, 3]$?

Solution:

The particle moves to the right when $0 < t < 1$ and to the left when $1 < t < 3$	**2 points: Process to compute total distance**
Total distance $= \lvert s(0) - s(1)\rvert + \lvert s(1) - s(3)\rvert = \lvert 0 - \frac{8}{3}\rvert + \lvert\frac{8}{3} - 0\rvert = \frac{16}{3}$	**2 points: Correct answer**

Award **2 points** if a student computes net distance $s(3) - s(0)$.

(c) (2 points) Find the acceleration $a(t)$ of the particle.

Solution:

$a(t) = v'(t) = s''(t) = 4t - 8$	**2 points: Correct answer (give full credit for correct answer with no work)**

Award **1 point** if a student recognizes that $a(t) = v'(t)$ or $a(t) = s''(t)$ but does not correctly compute $a(t)$.

Free Response Questions: Show your work!

(13) (10 points) Consider the curve described by the equation

$$\sin(y^2e^y) = x^2 + y - 1.$$

Find an equation of the line tangent to this curve at the point $(-1, 0)$.

Solution:

$$\begin{aligned}
\frac{d}{dx}\left(\sin(y^2e^y)\right) &= \frac{d}{dx}\left(x^2 + y - 1\right) \\
\cos(y^2e^y)\frac{d}{dx}\left(y^2e^y\right) &= 2x + \frac{dy}{dx} \\
\cos(y^2e^y)\left(y^2e^y\frac{dy}{dx} + 2y\frac{dy}{dx}e^y\right) &= 2x + \frac{dy}{dx}
\end{aligned}$$

6 points: Compute derivative

At $(-1, 0)$, the equation above reduces to $0 = -2 + \frac{dy}{dx}$. Thus,

$$\left.\frac{dy}{dx}\right|_{(-1,0)} = 2.$$

2 points: Find $\frac{dy}{dx}$ at $(-1, 0)$

Slope of the line is 2, the equation for the line is $y - 0 = 2(x + 1)$ or $y = 2x + 2$.

2 points: Equation of the tangent line at the point $(-1, 0)$

Free Response Questions: Show your work!

(14) (a) (5 points) **Use the limit definition** to compute $f'(x)$ if $f(x) = 1 - x^{-1}$.

Solution:

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{1-(x+h)^{-1}-(1-x^{-1})}{h} \end{aligned}$$

3 points: Definition of the derivative and difference quotient

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{\frac{1}{x+h}-\frac{1}{x}}{h} \\ &= \lim_{h\to 0} \frac{x-(x+h)}{(x+h)xh} \\ &= \lim_{h\to 0} \frac{1}{(x+h)x} = \frac{1}{x^2} \end{aligned}$$

2 points: Calculate correct limit

Award **1 point** if a student finds the answer using differentiation rules.

(b) (5 points) **Use differentiation rules** to compute $g''(x)$ if $g(x) = \ln(3x^2+x+1)$. You do not need to simplify your answer.

Solution:

$$\begin{aligned} g'(x) &= \frac{1}{3x^2+x+1}\frac{d}{dx}\left(3x^2+x+1\right) \\ &= \frac{6x+1}{3x^2+x+1} \end{aligned}$$

3 points: Find $g'(x)$ (chain rule, ln **rule, and power rule)**

$$\begin{aligned} g''(x) &= \frac{6(3x^2+x+1)-(6x+1)(6x+1)}{(3x^2+x+1)^2} \\ &= \frac{6}{3x^2+x+1} + \frac{(6x+1)^2}{(3x^2+x+1)^2} \end{aligned}$$

2 points: Find $g''(x)$ (quotient rule, power rule)

Free Response Questions: Show your work!

(15) A runner is jogging due east at 9 kilometers per hour. A coffee shop is located 4 kilometers due north of her starting point. How fast is the distance between the runner and the coffee shop increasing when the runner is 5 kilometers away from the coffee shop? Draw a picture of the situation, which includes the relevant quantities, and include units in your final answer.

$x = x(t)$: distance of the runner from her starting point

$y = y(t)$: distance from the runner to the coffee shop

$\frac{dx}{dt} = 9$ km/h

Find $\frac{dy}{dt}$ when $y = 5$.

3 points: Picture of situation and variables

By Pythagorean Theorem, $4^2 + x^2 = y^2$

1 point: Equation connecting variables

$\frac{d}{dt}\left(4^2 + x^2\right) = \frac{d}{dt}\left(y^2\right)$

$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$

$\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt}$

2 points: Differentiate

When $y = 5$, $x = 3$ by the equation $4^2 + x^2 = y^2$

1 point: Find x when $y = 5$

$\frac{dy}{dt} = \frac{3}{5}(9) = \frac{27}{5}$ km/h

3 points: Solve for $\frac{dy}{dt}$, 1 point is for units in the final answer

2013-S-2