MA 113 Calculus I — Spring 2014

Exam 2 — Tuesday, 11 March 2014

Name: ______________________

Section: ______________________

Last 4 digits of student ID #: ________

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box.*

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

[C,C,B,C,A, B,D,B,D,A]

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. Suppose $f(x) = 2xe^x$ and $g(x) = \cos x$. Which formula gives $g(f(x))$?

   (A) $2xe^x \cos x$

   (B) $2\cos(xe^x)$

   (C) $\cos(2xe^x)$

   (D) $2xe^{x\cos x}$

   (E) $2x\cos(e^x)$

2. A feather is dropped from 125 meters above the ground and falls with a constant acceleration of 10 meters/second^2 in the downward direction. After how many seconds does the feather hit the ground?

   (A) 25 seconds

   (B) 10 seconds

   (C) 5 seconds

   (D) 1 seconds

   (E) none of the above

Record the correct answer to the following problems on the front page of this exam.

3. The derivative of the function $f(x) = e^x \tan x$ is

   (A) $e^x \sec^2 x$

   (B) $e^x(\sec^2 x + \tan x)$

   (C) $e^x \sec x \tan x$

   (D) $e^x(\sec x \tan x + 1)$

   (E) $e^x(\sec^2 x + 1)$

4. If $g(x) = x^3 + 4x^2 + 2x$, what is $g''(x)$?

   (A) $g''(x) = 6x$

   (B) $g''(x) = 3x + 4$

   (C) $g''(x) = 6x + 8$

   (D) $g''(x) = 3x^2 + 8x + 2$

   (E) $g''(x) = x$

Record the correct answer to the following problems on the front page of this exam.

5. If $h(x) = \tan^{-1}(e^{x^2})$, what is $h'(x)$?

   (A) $2xe^{x^2}(1 + e^{2x^2})^{-1}$

   (B) $e^{x^2}(1 + e^{2x^2})^{-1}$

   (C) $x^2e^{x^2}(1 + e^{2x^2})^{-1}$

   (D) $xe^{x^2}(1 + e^{x^2})^{-1}$

   (E) $2xe^{x^2}(1 + e^{x^2})^{-2}$

6. Suppose that f has an inverse function f^{-1}. We know that $f(4) = 5$, $f(5) = 6$, $f'(4) = 2$, and $f'(5) = 1$. Find the derivative of f^{-1} at $x = 5$.

   (A) 1/5

   (B) 1/2

   (C) 2

   (D) 5

   (E) 2/5

Record the correct answer to the following problems on the front page of this exam.

7. Find the number r so that $y(x) = 20e^{rx}$ satisfies $y' = -2y$.

   (A) $r = 10$

   (B) $r = -10$

   (C) $r = 2$

   (D) $r = -2$

   (E) $r = 1$

8. Suppose a function $y(x)$ satisfies $y^2 + xy = e^x$. If $y(0) = -1$, what is $y'(0)$?

   (A) -2

   (B) -1

   (C) 0

   (D) 2

   (E) 1

Record the correct answer to the following problems on the front page of this exam.

9. The pressure P and volume V of an expanding gas are related by the equation $PV^b = C$, where b and C are nonzero constants. The volume is changing as a function of time. The rate for change of the pressure with respect to time, $P'(t)$, is:

   (A) $P'(t) = \frac{P}{V}V'(t)$

   (B) $P'(t) = b(PV)V'(t)$

   (C) $P'(t) = \frac{P}{V}V'(t)$

   (D) $P'(t) = -b\frac{P}{V}V'(t)$

   (E) $P'(t) = b\frac{P}{V}V'(t)$

10. The equation of the line tangent to the graph of the function $f(x) = \ln(2x^2 + 1)$ at the point $x = 1$ is

    (A) $y(x) = \frac{4}{3}(x - 1) + \ln 3$

    (B) $y(x) = \frac{1}{3}(x - 1) + 3$

    (C) $y(x) = \frac{4}{9}(x + 1) + \ln 3$

    (D) $y(x) = \frac{4}{3}x + \ln 3$

    (E) $y(x) = \frac{4}{3}(x + 1) + 3$

Free Response Questions: Show your work!

11. A rocket travels vertically with a constant speed of 10 kilometers/second. It is tracked by a telescope located 10 kilometers from the launch pad. Both the launch pad and the telescope are on the ground.

    (a) Carefully sketch the situation labeling all important aspects.

    (b) Find the rate (in radians per second) at which the angle between the telescope and the ground is increasing 3 seconds after lift off.

(a)	3 points total 1 point each for: angle between telescope and ground, height of rocket, distance between rocket lift off point and telescope
(b)	7 points total
Let $\theta = \theta(t)$ denote the angle between the telescope and the ground. Let $h = h(t)$ denote the height of the rocket. Then $$\tan(\theta(t)) = \frac{h(t)}{10}$$ OR $$\theta(t) = \tan^{-1}\left(\frac{h(t)}{10}\right)$$	2 points: Correct relationship between the variables
Differentiate both sides with respect to t: $$\sec^2(\theta(t))\frac{d\theta}{dt} = \frac{1}{10}\frac{dh}{dt}$$ OR $$\frac{d\theta}{dt} = \frac{1}{1+(\frac{h(t)}{10})^2}\cdot\frac{\frac{dh}{dt}}{10}$$	2 points: Correctly differentiate with respect to t
$\frac{dh}{dt} = 10$ kilometers/second, $\cos^2\big(\theta(3)\big) = 1/10$, $h(3) = 30$ kilometers	2 points: Correctly find values for all quantities in the equation above
$\frac{d\theta}{dt}\Big\rvert_{t=3} = \frac{1}{10}$ radians / second	1 point: Correct final answer

12. (a) State carefully the definition of the derivative of a function f at a point x in its domain using the concept of the limit.

    (b) Use the definition of the derivative in part (a) to compute the derivative of $f(x) = \dfrac{1}{1+x}$ at $x = 2$.

(a) The derivative of f at x is $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h},$$ provided the limit exists.	3 points total : 2 points: Correct difference quotient 1 point for stating "if the limit exists"
(b) By part (a), $$\begin{aligned} &\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} \\ &= \lim_{h\to 0} \frac{\frac{1}{1+(2+h)} - \frac{1}{1+2}}{h} \\ &= \lim_{h\to 0} \frac{3-(3+h)}{h(3+h)(3)} \\ &= \lim_{h\to 0} \frac{-1}{(3+h)(3)} \\ &= -\frac{1}{9}. \end{aligned}$$ The limit above exists and is equal to $-1/9$, therefore $f'(2) = -1/9$.	7 points total: 2 points: Correct difference quotient 3 points: Correct algebra to simplify difference quotient and to cancel h from top and bottom 2 points: Correct evaluation of limit and final answer Award only 1 point for a bald answer or for calculating the derivative using differentiation rules.

Free Response Questions: Show your work!

13. Find the first derivative of the following functions:

    (a) $f(x) = \ln(2x^2 + x)$, for $x > 0$

    (b) $g(x) = \cos^{-1}(4x^2)$, for $|x| < 1/2$

(a) $f'(x) = \dfrac{1}{2x^2+x} \cdot (4x+1)$ $f'(x) = \dfrac{4x+1}{2x^2+x}$	5 points total: 2 point chain rule 2 points derivative of ln 1 points derivative of $2x^2 + x$
(b) $g'(x) = \dfrac{-1}{\sqrt{1-(4x^2)^2}} \cdot (8x)$ $g'(x) = \dfrac{-8x}{\sqrt{1-16x^4}}$	5 points total: 2 points chain rule 2 points derivative of $\cos^{-1}$ 1 point derivative of $4x^2$

Derivatives do not need to be simplified.

14. Suppose $y(x)$ satisfies the relation

$$x^2 + xy - y^2 = 1$$

(a) Find a formula for $y'(x)$ that may involve x and y.

(b) Using the formula from part (a), find y' at $(x, y) = (2, 3)$.

(c) Find the equation of the tangent line to this curve at the point $(2, 3)$. Write the equation in the form $y = mx + b$.

(a) Take d/dx of both sides: $$\begin{aligned} \frac{d}{dx}\left(x^2 + xy - y^2\right) &= \frac{d}{dx}(1) \\ 2x + (y + xy'(x)) - 2yy'(x) &= 0 \\ y'(x)\left(x - 2y\right) &= -2x - y \\ y'(x) &= \frac{2x + y}{2y - x} \end{aligned}$$	6 points total: 3 points: Correct differentiation 3 points: Correct expression for $y'(x)$
(b) Evaluate $y'(x)$ at $(2, 3)$: $y'(x)\Big\vert_{(2,3)} = \dfrac{2 \cdot 2 + 3}{2 \cdot 3 - 2} = \dfrac{7}{4}$	2 points
(c) $y - 3 = \dfrac{7}{4}(x - 2)$ $y = \frac{7}{4}x - \frac{1}{2}$	2 points

Free Response Questions: Show your work!

15. The linear motion of a body is given by $s(t) = s_0 + v_0 t - \frac{1}{2}gt^2$, where $g = 9.8$ meters/second^2. A girl launches a toy rocket from the ground that flies straight upward. It has an initial velocity of 19.6 meters/second.

    (a) What is the maximal height attained by the rocket?

    (b) After how many seconds does the rocket reach its maximal height?

    (c) After how many seconds will the rocket return to the ground?

(a) The equation of motion of the rocket is $$s(t) = 19.6t - \frac{9.8}{2}t^2.$$ Maximal height is reached when velocity is 0. Solve for t when $v(t) = s'(t) = 0$: $$0 = 19.6 - 9.8t,$$ Maximal height occurs at $t = 2$ seconds. Then the maximum height is $$s(2) = 19.6(2) - \frac{9.8}{2}4 = 19.6 \text{ meters}$$	4 points total: 3 points: Correct method to find maximal height 1 point: Correct final answer
(b) Maximal height is reached at $t = 2$ seconds by part (a)	3 points
(c) Solve $s(t) = 0$: $$19.6t - \frac{9.8}{2}t^2 = 0$$ $$t(19.6 - \frac{9.8}{2}t) = 0.$$ Then $t = 0$ or $t = 4$. The rocket returns to the ground at $t = 4$ seconds.	3 points total: 2 points: Set $s(t) = 0$ and solve for t 1 point: Correct final answer