MA 113 Calculus I — Spring 2018

Exam 2 — Tuesday, 6 March 2018

Name: ______________________________

Section: ____________________________

Last 4 digits of student ID #: ________

This exam has 12 multiple choice questions (five points each) and 4 free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-buds during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box.*

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E

[B,E,D,C,A,E, C,B,D,E,A,D]

Exam Scores

Question	Score	Total
MC		60
13		10
14		10
15		10
16		10
Total		100

1. Suppose that $h(x) = f(g(x))$. Suppose that $h'(1) = 3$, $g'(1) = 5$, and $g(1) = 7$. Find $f'(7)$.

   (A) $1/7$

   (B) $3/5$

   (C) $5/7$

   (D) $7/3$

   (E) None of the above

2. Let $g(x) = 3x^{-4} - 7\sqrt{x} + 4x^{-2/5}$. Find $g'(x)$.

   (A) $-12x^{-5} - \dfrac{7}{2\sqrt{x}} - \dfrac{8}{5}x^{3/5}$

   (B) $-12x^{-5} - \dfrac{7}{\sqrt{x}} - \dfrac{8}{5}x^{-7/5}$

   (C) $-12x^{-3} - \dfrac{7}{\sqrt{x}} - \dfrac{8}{5}x^{-7/5}$

   (D) $-12x^{-3} - \dfrac{7}{2\sqrt{x}} - \dfrac{8}{5}x^{3/5}$

   (E) $-12x^{-5} - \dfrac{7}{2\sqrt{x}} - \dfrac{8}{5}x^{-7/5}$

Record the correct answer to the following problems on the front page of this exam.

3. Let f be defined by

$$f(x) = \begin{cases} x^2 - 3x, & \text{if } x \geq 4 \\ Ax + B & \text{if } x < 4. \end{cases}$$

Find the values of A and B that make f differentiable everywhere.

(A) $A = 2$ and $B = 4$

(B) $A = 2$ and $B = 12$

(C) $A = 5$ and $B = 4$

(D) $A = 5$ and $B = -16$

(E) None of the above

4. The expression $\lim\limits_{h \to 0} \dfrac{\tan(\frac{\pi}{4} + h) - 1}{h}$ represents a derivative $f'(a)$. Find $f(x)$, find a, and find the limit.

(A) $f(x) = \sec^2(x)$, $a = 0$, and the limit equals 0

(B) $f(x) = \tan(x)$, $a = 0$, and the limit equals 1

(C) $f(x) = \tan(x)$, $a = \frac{\pi}{4}$, and the limit equals 2

(D) $f(x) = \sec^2(x)$, $a = 0$, and the limit equals 2

(E) $f(x) = \tan(x)$, $a = \frac{\pi}{4}$, and the limit equals 0

Record the correct answer to the following problems on the front page of this exam.

5. Find the equation of the tangent line to the graph of $y = \ln(x^2)$ at $x = e^3$.

   (A) $y = \dfrac{2}{e^3}x + 4$

   (B) $y = \dfrac{2}{e^6}x + 6 - \dfrac{2}{e^3}$

   (C) $y = \dfrac{2}{e^3}x + 9$

   (D) $y = \dfrac{2}{e^6}x + 9 - \dfrac{2}{e^3}$

   (E) $y = \dfrac{2}{e^3}x + 7$

6. Let $f(x) = e^{x^2}$. Find $f''(x)$.

   (A) $(2x^2 + 2)e^{x^2}$

   (B) $(2x + 2)e^{x^2}$

   (C) $(4x + 2)e^{x^2}$

   (D) $2xe^{x^2}$

   (E) $(4x^2 + 2)e^{x^2}$

Record the correct answer to the following problems on the front page of this exam.

7. Let $f(x) = \sin^2(x) + \cos(3x)$. Find the second derivative $f''(x)$.

   (A) $2(\cos^2(x) - \sin^2(x)) + 9\cos(3x)$

   (B) $2(\cos^2(x) - \sin^2(x)) - 3\cos(3x)$

   (C) $2(\cos^2(x) - \sin^2(x)) - 9\cos(3x)$

   (D) $2(-\cos^2(x) + \sin^2(x)) - 9\cos(3x)$

   (E) $2(-\cos^2(x) + \sin^2(x)) - 3\cos(3x)$

8. Suppose that $h(x) = \tan^4(2x)$. Find $h'(x)$.

   (A) $4\tan^3(2x)(2)\sec^2(2x)(2)$

   (B) $4\tan^3(2x)\sec^2(2x)(2)$

   (C) $4\tan^3(2x)\sec^2(2x)$

   (D) $4\tan^3(2x)(2)$

   (E) $4\tan^3(2x)$

Record the correct answer to the following problems on the front page of this exam.

9. Let $f(x) = \ln(x^4 + 3x^2 + 9)$. Find $f'(2)$.

   (A) 1/13

   (B) 1/37

   (C) 10/13

   (D) 44/37

   (E) 44/29

10. Let $f(x) = \dfrac{e^{2x}}{\sqrt{x}}$. Find $f'(4)$.

   (A) $\dfrac{3}{8}e^8$

   (B) $\dfrac{7}{16}e^8$

   (C) $\dfrac{9}{16}e^8$

   (D) $\dfrac{13}{16}e^8$

   (E) $\dfrac{15}{16}e^8$

Record the correct answer to the following problems on the front page of this exam.

11. The length of a rectangle is increasing at a rate of 2 cm/s and its width is increasing at a rate of 3 cm/s. How fast is the area of the rectangle increasing when the length is 4 cm and the width is 5 cm?

    (A) 22 cm^2/s
    (B) 23 cm^2/s
    (C) 25 cm^2/s
    (D) 26 cm^2/s
    (E) 27 cm^2/s

12. A particle is moving along a hyperbola given by $xy = 8$. At the point $(4, 2)$, the y-coordinate of the particle satisfies $\dfrac{dy}{dt} = 3$ cm/s. Find $\dfrac{dx}{dt}$ at the point $(4, 2)$.

    (A) 4 cm/s
    (B) -4 cm/s
    (C) 6 cm/s
    (D) -6 cm/s
    (E) None of the above

Free Response Questions: Show your work!

13. Find the limits or state that the limit does not exist. In each case, justify your answer.

    (Students who guess the answer based on a few values of the function will not receive full credit.)

    (a) $\displaystyle\lim_{x\to 0} \frac{\sin(2x)\tan(2x)}{x^2}$

    $$\lim_{x\to 0} \frac{\sin(2x)\tan(2x)}{x^2} = \lim_{x\to 0} \frac{\sin(2x)}{2x}\frac{\sin(2x)}{2x}\frac{4}{\cos(2x)} = 1 \cdot 1 \cdot \frac{4}{1} = 4.$$

    (b) $\displaystyle\lim_{x\to 0} \frac{2x^3 + 7x}{\sin(3x)}$

$$\lim_{x\to 0} \frac{2x^3 + 7x}{\sin(3x)} = \lim_{x\to 0} \frac{2x^2}{3} \cdot \frac{3x}{\sin(3x)} + \frac{7}{3}\frac{3x}{\sin(3x)} = 0 \cdot 1 + \frac{7}{3} \cdot 1 = \frac{7}{3}.$$

14. A man of height 2 meters walks away from a 7.2-meter lamppost at a speed of 1.3 meters per second. Find the rate at which his shadow is increasing in length.

    Let x be the distance of the man from the lamppost and let L be the length of the man's shadow. By similar triangles we have $\dfrac{x+L}{7.2} = \dfrac{L}{2}$. Then $2x + 2L = (7.2)L$, which gives $2x = (5.2)L$. Thus $L = \dfrac{2}{5.2}x$. Differentiate with respect to t to get $\dfrac{dL}{dt} = \dfrac{2}{5.2}\dfrac{dx}{dt} = \dfrac{2}{5.2} \cdot (1.3) = .5$. Thus the rate at which the shadow is increasing in length is .5 meters per second.

15. (a) Use implicit differentiation to find the slope of the tangent line to the curve $y\sin(3x) = x\cos(3y)$ at the point $(\pi/3, \pi/6)$.

$$y\cos(3x)\cdot 3 + \sin(3x)\frac{dy}{dx} = -x\sin(3y)\cdot 3\frac{dy}{dx} + \cos(3y)$$
$$(\sin(3x) + 3x\sin(3y))\frac{dy}{dx} = \cos(3y) - 3y\cos(3x)$$
$$\frac{dy}{dx} = \frac{\cos(3y) - 3y\cos(3x)}{\sin(3x) + 3x\sin(3y)}$$
$$\frac{dy}{dx}(\pi/3, \pi/6) = \frac{0 - \frac{\pi}{2}\cdot(-1)}{0 + \pi\cdot 1} = \frac{1}{2}$$

(b) Find the equation of the tangent line to the curve $y\sin(3x) = x\cos(3y)$ at the point $(\pi/3, \pi/6)$.

$$y - \frac{\pi}{6} = \frac{1}{2}(x - \frac{\pi}{3}),\ y = \frac{1}{2}x$$

Free Response Questions: Show your work!

16. The height in feet of a projectile shot vertically upward from a point 96 feet above ground level with an initial velocity of 80 feet per second is $h(t) = 96 + 80t - 16t^2$ after t seconds. (To receive credit, you must justify your work.))

    (a) When does the projectile reach its maximum height?

    The projectile reaches its maximum height when $h'(t) = v(t) = 0$. Thus $-32t + 80 = 0$ gives $t = \dfrac{5}{2}$ seconds.

    (b) What is the maximum height?

    The maximum height is $h(5/2) = 196$ feet.

    (c) When does the projectile hit the ground?

    The projectile hits the ground when $h(t) = 0$. This gives $-16(t^2 - 5t - 6) = -16(t - 6)(t + 1) = 0$, and so $t = 6, -1$. Since $t > 0$, we have $t = 6$ seconds.

    (d) With what velocity does the projectile hit the ground?

    The velocity when the projectile hits the ground is $v(6) = h'(6) = -32 \cdot 6 + 80 = -112$ feet per second.