MA 113 Calculus I — Spring 2019

Exam 2 — Tuesday, 5 March 2019

Name: __________________________

Section: _________________________

Last 4 digits of student ID #: ________

This is a two-hour exam. This exam has 12 multiple choice questions (five points each) and 4 free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-buds during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.

- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box.*

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,

- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

Question						
1	A	B	C	D	E	C
2	A	B	C	D	E	A
3	A	B	C	D	E	B
4	A	B	C	D	E	C
5	A	B	C	D	E	D
6	A	B	C	D	E	A
7	A	B	C	D	E	A
8	A	B	C	D	E	C
9	A	B	C	D	E	B
10	A	B	C	D	E	A
11	A	B	C	D	E	B
12	A	B	C	D	E	E

Exam Scores

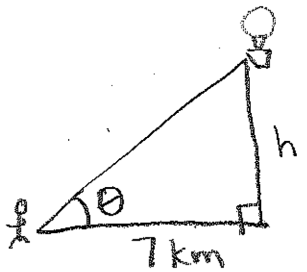
Question	Score	Total
MC		60
13		10
14		10
15		10
16		10
Total		100

*KEY * KEY * KEY * KEY *

Free Response Questions: Show your work!

13. A hot air balloon rising vertically is tracked by an observer located 7 km from the liftoff point.

(3pts) (a) Find an equation to relate the height of the balloon and the angle of the observer's line-of-sight.



(+1) picture

$\tan\theta = \frac{opp}{adj}$

$$\boxed{\tan\theta = \frac{h}{7}}$$

(+1) (+1)

(7pts) (b) At a certain moment, the angle between the observer's line-of-sight and the horizontal is $\frac{\pi}{6}$ and it is changing at a rate of 0.2 radians per minute. How fast is the balloon rising at that moment? Include units!!!

Given

$\theta = \frac{\pi}{6}$

$\frac{d\theta}{dt} = 0.2$

$\sec^2\left(\frac{\pi}{6}\right) = \left(\frac{1}{\cos\frac{\pi}{6}}\right)^2 = \left(\frac{1}{\sqrt{3}/2}\right)^2 = \frac{4}{3}$

differentiate!

$\sec^2\theta \, \frac{d\theta}{dt} = \frac{1}{7}\frac{dh}{dt}$ (+2) one point each side

$\sec^2\left(\frac{\pi}{6}\right)(0.2) = \frac{1}{7}\frac{dh}{dt}$ (+2) plugging in θ and $\frac{d\theta}{dt}$

$\left(\frac{4}{3}\right)(0.2) = \frac{1}{7}\frac{dh}{dt}$ (+1) $\sec^2\left(\frac{\pi}{6}\right) = \frac{4}{3}$

$\left(\frac{4}{3}\right)(0.2)(7) = \frac{dh}{dt}$

$\frac{dh}{dt} = \boxed{1.867 \text{ km/min}}$

(+1) numerical answer (+1) units

16. This problem concerns the definition of the derivative using limits.

(a) State the formal definition of the derivative of a function $f(x)$ at the point $x = a$.
Hint: Your definition should involve a limit.

4 pts

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

+1 +1 +1 +1

Same scheme if using this form.

6 pts

(b) **Using the formal definition of derivative and the limit laws**, find the derivative of the function $f(x) = \frac{1}{3x}$. An answer that is unsupported or uses differentiation rules will receive **no** credit.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \boxed{\lim_{h \to 0} \frac{\frac{1}{3(x+h)} - \frac{1}{3x}}{h}}$$

← +2 for writing this.

$$= \lim_{h \to 0} \frac{\frac{x - (x+h)}{3(x+h)x}}{h} = \frac{1}{3} \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}(x+h)x} = \boxed{\frac{1}{3} \cdot \frac{-1}{x^2}}.$$

+2 for common denominator

+1 for canceling h

+1 for final answer

Spring 2009 M. Reddy / V. Lawrence / M. von Bell.

Free Response Questions: Show your work!

14. (a) Find the equation of the tangent line to $4x^6 + y^6 = 5$ at the point $(-1, 1)$.

$4x^6 + y^6 = 5$

Differentiate wrt x:

1 point for "0"

1 point for dy/dx

$$24x^5 + 6y^5 \frac{dy}{dx} = 0$$

1 point for "24x^5" and "6y^5"

$$\frac{dy}{dx} = \frac{-24x^5}{6y^5} = \frac{-4x^5}{y^5}$$

At $(-1,1)$ slope of tangent is

$$m = \frac{-4(-1)^5}{(1)^5} = +4$$

1 point for slope

1 point for eqn of line

So eqn of tangent @ $(1,-1)$ is

$y - 1 = 4(x+1)$

or

$y = 4x + 5.$

(b) Find $\lim_{x \to 0} \frac{\sin^2(2x)}{5x^2}$.

1 point for splitting sin^2(2x) as sin(2x) sin(2x)

$$\lim_{x \to 0} \frac{1}{\frac{5}{4}} \cdot \frac{\sin(2x)}{2x} \cdot \frac{\sin(2x)}{2x}$$

1 point for dividing each sin term by 2x

1 point for correct constant here

$$= \lim_{x \to 0} \frac{4}{5} \cdot \frac{\sin(2x)}{2x} \cdot \frac{\sin(2x)}{2x}.$$

Let $\theta = 2x$.
As $x \to 0$, $\theta \to 0$.

1 point for limit sin(th)/th=1 as th->0

So

$$= \lim_{\theta \to 0} \frac{4}{5} \frac{\sin\theta}{\theta} \cdot \frac{\sin\theta}{\theta} = \frac{4}{5}\left(\lim_{\theta \to 0} \frac{\sin\theta}{\theta}\right)\left(\lim_{\theta \to 0} \frac{\sin\theta}{\theta}\right)$$

$$= \frac{4}{5} \cdot 1 \cdot 1 = 4/5.$$

1 point for sliding constant through limit

Sol'n #2

14. a. $4x^6 + y^6 = 5$

$$y = +(5-4x^6)^{1/6}$$

$$\frac{dy}{dx} = \frac{1}{6}(5-4x^6)^{-5/6} \cdot (-24x^5).$$

At $(-1,+1)$

$$m = \frac{1}{6}(1)^{-5/6} \cdot (+24)(1) = 4$$

So

$$y-1 = 4(x+1)$$

or

$$y = 4x+5$$

14b. Let $\theta = 2x$. $(x = \theta/2)$ As $x \to 0$, $\theta \to 0$.

So

$$= \lim_{\theta \to 0} \frac{\sin^2\theta}{5 \cdot \left(\frac{\theta}{2}\right)^2}$$

$$= \lim_{\theta \to 0} \frac{\sin^2\theta}{\frac{5}{4}\theta^2} = \frac{4}{5} \lim_{\theta \to 0} \frac{\sin^2\theta}{\theta^2}$$

$$= \frac{4}{5} \lim_{\theta \to 0} \left(\frac{\sin\theta}{\theta}\right)^2 = \frac{4}{5}(1)^2 = \frac{4}{5}.$$

Free Response Questions: Show your work!

15. Let $f(x) = x \cdot \ln(x^4 - x + e^2)$.

(a) Find the derivative $f'(x)$.

$$f'(x) = \ln(x^4 - x + e^2) + x \cdot \frac{4x^3 - 1}{x^4 - x + e^2}$$

Correct use of product rule	2 points (1 for only writing formula)
$\frac{d}{du} \ln u = \frac{1}{u}$	1 point
Use of chain rule for $\frac{d}{dx} \ln(x^4 - x + e^2)$	1 point
Correct derivatives of x and $x^4 - x + e^2$	1 point

(b) Find the equation of the tangent line to $f(x)$ at the point where $x = 1$.

Slope of tangent line at $x = 1$:

$$f'(1) = \ln(1 - 1 + e^2) + 1 \cdot \frac{4 - 1}{1 - 1 + e^2} = 2 + \frac{3}{e^2}$$

y-coordinate of function and tangent line at $x = 1$:

$$f(1) = 1 \cdot \ln(1 - 1 + e^2) = 2$$

Equation of tangent line to $f(x)$ at $x = 1$:

$$y - 2 = \left(2 + \frac{3}{e^2}\right)(x - 1), \text{ or}$$

$$y = \left(2 + \frac{3}{e^2}\right)x - \frac{3}{e^2}$$

Attempted to find $f'(1)$	1 point
$f'(1)$ correct	1 point (OK if not fully simplified)
$f(1)$ correct	1 point (OK if not fully simplified)
Tangent line equation with 1, $f(1)$, $f'(1)$ all correctly placed	2 points (partial credit possible)