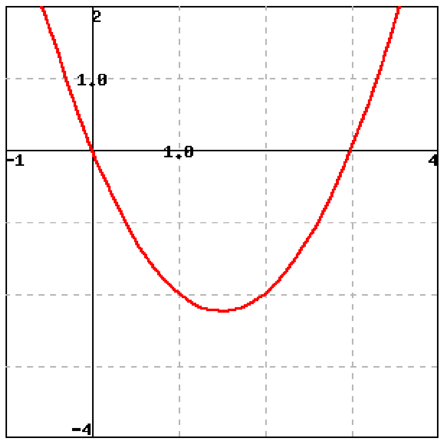
Exam 2

Solutions

Multiple Choice Questions

1. Calculate the slope of the secant line through the points on the graph where $x = 1$ and $x = 3$.



A. -1

B. 2

C. 0

D. 1

E. -2

2. Find $f(3)$ and $f'(3)$, assuming that the tangent line to $y = f(x)$ at $x = 3$ has equation $y = 3x - 2$.

A. $f(3) = 2, f'(3) = 3$

B. $f(3) = 7, f'(3) = 3$

C. $f(3) = -2, f'(3) = 3$

D. $f(3) = 3, f'(3) = 2$

E. $f(3) = 9, f'(3) = 2$

3. Determine coefficients a and b such that $p(x) = x^2 + ax + b$ satisfies $p(1) = 11$ and $p'(1) = 11$.

   A. $a = 1, b = 9$

   B. $a = 10, b = 0$

   C. $a = 0, b = 10$

   D. $a = 8, b = 3$

   E. $a = 9$**,** $b = 1$

4. The length of a rectangle is increasing at a rate of 8 cm/sec and its width is increasing at a rate of 3 cm/sec. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing?

   A. 190 cm^2/sec

   B. 224 cm^2/sec

   C. 140 **cm^2/sec**

   D. 24 cm^2/sec

   E. 200 cm^2/sec

5. Find a formula for $\dfrac{dy}{dx}$ in terms of x and y, where $x^4y + 4xy^4 = x + y$.

   A. $\dfrac{dy}{dx} = \dfrac{1 - x^4y - 16xy^3}{4x^3y + 4y^4 - 1}$

   B. $\dfrac{dy}{dx} = \dfrac{-1}{1 - 4x^3 - 16y^3}$

   C. $\dfrac{dy}{dx} = 4x^3y + x^4 + 4y^4 + 4x - 1$

   D. $\dfrac{dy}{dx} = \dfrac{4x^3y + 4y^4 - 1}{1 - x^4 - 16xy^3}$

   E. $\dfrac{dy}{dx} = \dfrac{-1}{1 - x^4 - 16x}$

6. Find $f'''(x)$ where $f(x) = xe^x$.

   A. $f'''(x) = e^x$

   B. $f'''(x) = (x+3)e^x$

   C. $f'''(x) = (x+1)e^x$

   D. $f'''(x) = (x+2)e^x$

   E. $f'''(x) = 3xe^x$

7. Find $f'(x)$ in terms of $g'(x)$ where $f(x) = x^2[g(x)]^2$.

   A. $f'(x) = 2x[g(x)]^2 + 2x^2 g(x)g'(x)$

   B. $f'(x) = 4xg'(x)$

   C. $f'(x) = 2x[g'(x)]^2$

   D. $f'(x) = 2x[g(x)]^2 + x^2[g'(x)]^2$

   E. $f'(x) = 4xg(x)g'(x)$

8. Find the derivative of $g(x) = x \arctan(x)$. (Remember that $\arctan(x)$ is the same as $\tan^{-1}(x)$.)

   A. $g'(x) = \dfrac{1}{1+x^2}$

   B. $g'(x) = \dfrac{x}{1+x^2}$

   C. $g'(x) = \arctan(x) + \dfrac{x}{1+x^2}$

   D. $g'(x) = \arctan(x) + \dfrac{1}{1+x^2}$

   E. $g'(x) = \arctan(x)$

9. Find the derivative of

$$h(x) = \frac{\ln(x^2)}{x^5}.$$

A. $h'(x) = \dfrac{1}{5x^6}$

B. $h'(x) = \dfrac{2}{5x^5}$

C. $h'(x) = \dfrac{1 - 5x\ln(x^2)}{x^7}$

D. $h'(x) = \dfrac{2 - 5\ln(x^2)}{x^6}$

E. $h'(x) = \dfrac{5\ln(x^2) - 2}{x^6}$

10. Differentiate

$$f(x) = \frac{\cos(2x)}{1 - x^2}$$

A. $f'(x) = \dfrac{\sin(2x)}{2x}$

B. $f'(x) = \dfrac{-2x\cos(2x) + 2(1 - x^2)\sin(2x)}{(1 - x^2)^2}$

C. $f'(x) = \dfrac{2(1 - x^2)\sin(2x) + 2x\cos(2x)}{(1 - x^2)^2}$

D. $f'(x) = \dfrac{-2(1 - x^2)\sin(2x) + 2x\cos(2x)}{1 - x^4}$

E. $f'(x) = \dfrac{-2(1 - x^2)\sin(2x) + 2x\cos(2x)}{(1 - x^2)^2}$

11. Suppose that $g(x) = \sin(x^2 - x - 6)$.
Find $g'(3)$

A. $\cos(5)$

B. 1

C. 0

D. 5

E. $\sin(5)$

12. The displacement (in meters) of a particle moving in a straight line is given by $s = 2t^2 - 6t + 5$, where t is measured in seconds. Find the average velocity over the time interval $[6, 8]$.

A. 63 m/sec

B. 4 m/sec

C. 44 m/sec

D. 8 m/sec

E. 22 **m/sec**

Free Response Questions
Show all of your work

13. Find the derivatives of the following functions.

(a) $f(x) = \ln(\tan(x))$.

Solution:
$$f'(x) = \frac{\sec^2(x)}{\tan(x)}$$

(b) $g(x) = \frac{4}{x^3} - \frac{6}{x^2} - \frac{8}{x} + 10$.

Solution: First, rewrite $g(x)$

$$\begin{aligned} g(x) &= 4x^{-3} - 6x^{-2} - 8x^{-1} + 10 \\ g'(x) &= -12x^{-4} + 12x^{-3} + 8x^{-2} \\ &= -\frac{12}{x^4} + \frac{12}{x^3} + \frac{8}{x^2} \end{aligned}$$

(c) $h(x) = 4\ln(x^2 e^{x^2})$.

Solution: First, rewrite $h(x)$

$$\begin{aligned} h(x) &= 4\ln(x^2) + 4\ln(e^{x^2}) \\ &= 8\ln x + 4x^2 \\ h'(x) &= \frac{8}{x} + 8x \end{aligned}$$

(d) $j(x) = \arcsin(2x)$

Solution:
$$j'(x) = \frac{2}{\sqrt{1-4x^2}}$$

14. (a) Find the equation of the tangent line to $y^2 = 5x^4 - x^2$ at the point $(1,2)$.

Solution: We need to find $\left.\dfrac{dy}{dx}\right|_{(1,2)}$.

$$\begin{aligned} y^2 &= 5x^4 - x^2 \\ 2y\frac{dy}{dx} &= 20x^3 - 2x \\ \frac{dy}{dx} &= \frac{10x^3 - x}{y} \end{aligned}$$

Now,

$$\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{10-1}{2} = \frac{9}{2}$$

So the equation of the tangent line is

$$y = 2 + \frac{9}{2}(x-1) = \frac{9}{2}x - \frac{5}{2}$$

(b) Find $\lim\limits_{x\to 0} \dfrac{\sin(3x)}{7x}$. (You may **NOT** use L'Hôpital's Rule to evaluate this.)

Solution:

$$\lim_{x\to 0} \frac{\sin(3x)}{7x} = \frac{3}{7} \lim_{x\to 0} \frac{\sin(3x)}{3x} = \frac{3}{7} \times 1 = \frac{3}{7}.$$

15. Let $f(x) = \dfrac{x^3}{x+8}$.

(a) Find the derivative $f'(x)$.

Solution: By the Quotient Rule

$$f'(x) = \frac{(x+8)(3x^2) - x^3(1)}{(x+8)^2} = \frac{2x^3 + 24x^2}{(x+8)^2}$$

(b) Find the equation of the tangent line to $f(x)$ at the point where $x = 2$.

Solution: $f(2) = 8/10 = 4/5$ and

$$f'(2) = \frac{2(8) + 24(4)}{100} = \frac{28}{25}.$$

Thus, the equation of the tangent line to the curve is

$$y = \frac{4}{5} + \frac{28}{25}(x-2) = \frac{28}{25}x - \frac{36}{25} = 1.12x - 1.44.$$

16. This problem concerns the definition of the derivative using limits.

    (a) State the formal definition of the derivative of a function $f(x)$ at the point $x = a$. *Hint*: Your definition should involve a limit.

    Solution: The derivative of the function $f(x)$ at the point $x = a$ is

    $$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

    if this limit exists.

    (b) **Using the formal definition of derivative and the limit laws**, find the derivative of the function $f(x) = x^2 + x - 1$. An answer that is unsupported or uses differentiation rules will receive **no credit**.

    Solution:

    $$\begin{aligned} f'(a) &= \lim_{h \to 0} \frac{(a+h)^2 + (a+h) - 1 - (2a^2 + a - 1)}{h} \\ &= \lim_{h \to 0} \frac{a^2 + 2ah + h^2 + a + h - 1 - a^2 - a + 1}{h} \\ &= \lim_{h \to 0} \frac{2ah + h^2 + h}{h} \\ &= \lim_{h \to 0} 2a + h + 1 \\ f'(a) &= 2a + 1 \end{aligned}$$