MA 113 — Calculus I Fall 2008
Exam 3 November 18, 2008

Answer all questions 1-7 and choose two of questions 8-10 to answer. Please indicate which of problems 8-10 is not to be graded by crossing through its number on the table below.

Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. All other electronic devices including pagers and cell phones should be in the off position for the duration of the exam.

Please:

1. **clearly indicate your answer and the reasoning used to arrive at that answer (unsupported answers may not receive credit),**
2. **give exact answers, rather than decimal approximations.**

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Name: KEY

Section: ______

Last four digits of student identification number: ______

Question	Score	Total
1		7
2		10
3		11
4		12
5		10
6		8
7		9
8		15
9		15
10		15
Free	3	3
		100

7

(1) Find the absolute maximum value of the function

$$f(x) = 2x^3 - 3x^2 - 12x + 1$$

on the interval $[-2, 3]$.

$f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2)$ 1

$= 6(x+1)(x-2)$

critical #'s $x = -1$, $x = 2$ 1

Test critical #'s and end pts

x	$f(x)$
-2	-3
-1	8
2	-19
3	-8

1 each 4

Absolute maximum $\underline{\quad 8 \quad}$ at $x = \underline{\quad -1 \quad}$ 1

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(2) Consider the function $f(x) = x^2 - 3x + \ln(x)$ on the interval $(0, \infty)$.

(a) Find the critical number(s) of f.

(b) Find the interval(s) of increase and decrease for f.

(c) Find the local extrema of f.

(a) $f'(x) = 2x - 3 + \frac{1}{x}$

$= \frac{1}{x}(2x^2 - 3x + 1)$

$= \frac{1}{x}(2x-1)(x-1)$ 2

Critical numbers $x = 1/2$ 1 $x = 1$ 1

(b) $\overset{+}{}\;0\;\overset{+}{}\;1/2\;\overset{-}{}\;1\;\overset{+}{}$

Show work 2

$f'(1/4) = 4(-1/2)(-3/4) > 0$

Answers 2

$f'(3/4) = 4/3(1/2)(-1/4) < 0$

$f'(2) = \frac{1}{2} \cdot 3 \cdot 1 > 0$

(c) Local max $f(1/2) = -5/4 + \ln(1/2)$ 1

$f(1) = 1 - 3 + \ln(1) = -2$ 1

(a) The critical number(s) $x = 1/2,\ x = 1$

(b) Interval(s) of increase $(0, 1/2)\ (1, \infty)$ and decrease $(1/2, 1)$

(c) The local maximum is at $x =$ $1/2$ and $f(x) =$ $-5/4 - \ln(2)$

The local minimum is at $x =$ 1 and $f(x) =$ -2

4

(3) Consider the function
$$f(x) = (x^2+1)e^{-x}.$$

(a) Find the interval(s) of concavity of the graph of $f(x)$ and show your work.

(b) Find the point(s) of inflection of the graph of $f(x)$ and justify your work.

(a) $f'(x) = (2x - x^2 - 1)e^{-x}$ 1

$$\left.\begin{aligned} f''(x) &= (x^2-2x+1)e^{-x} + (-2x+2)e^{-x} \\ &= (x^2-4x+3)e^{-x} \\ &= (x-1)(x-3)e^{-x} \end{aligned}\right\} \; 1$$

Sign f'' : $+$ on $(-\infty, 1)$, $-$ on $(1, 3)$, $+$ on $(3, \infty)$; test points $0, 2, 4$ 2 Sign graph or other work to show how intervals of concavity are obtained.

$f''(0) = 3 > 0$

$f''(2) = 1(-2)e^{-2} < 0$

$f''(4) = 3 \cdot 1 \, e^{-4} > 0$

$\therefore x = 1, \; x = 3$ are pts of inflection ← See below for answer scoring

(a) Interval(s) where graph is concave up $(-\infty, 1)$ 2 $(3, \infty)$ 2

Interval(s) where graph is concave down $(1, 3)$ 1

(b) Point(s) of inflection (x and y coordinates) $(1, 2e^{-1})$ 1 $(3, 10e^{-3})$ 1

6

(4) (a) Suppose that $f(x) = \sqrt{9+x}$.

- Find the linear approximation of $f(x)$ at $x = 0$.
- Use the linear approximation to estimate $\sqrt{9.05}$. Present your answer as a rational number.

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(b) Suppose that the line $y = 5x - 4$ is tangent to the graph of a function g at $x = 3$. If Newton's method is used to find a solution to the equation $g(x) = 0$, and the initial approximation is $x_1 = 3$, find the next approximation x_2.

1 1 1

(a) $f'(x) = \frac{1}{2\sqrt{9+x}}$ $\quad f(0) = 3 \quad$ $f'(0) = \frac{1}{2 \cdot 3} = \frac{1}{6}$

$L(x) = 3 + \frac{1}{6}(x-0) = 3 + \frac{1}{6}x$ 2

$L(0.05) = L(\frac{1}{20}) = 3 + \frac{1}{6} \cdot \frac{1}{20} = \frac{361}{120}$ (3.0083̅ as a decimal)

(b) Solve $L(x) = 0$ if $5x - 4 = L(x)$ 3 state method.

$5x - 4 = 0$ 2

$x = 4/5$ 1

OR

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ 2 $\quad f(x_0) = f(3) = 15 - 4 = 11$ 1

$f'(x_0) = 5$ 1

$x_1 = 3 - \frac{11}{5}$ 1

$= \frac{15-11}{5} = 4/5$ 1

(a) • Linear approximation $L(x) =$ $3 + \frac{1}{6}x$

• $\sqrt{9.05} \approx$ 3 61/120

(b) $x_2 =$ 4/5

(5) Find the general antiderivative of each function.

(a) $f(x) = x - 3$.

(b) $g(x) = 3e^x + 4\cos(x)$.

(c) $h(t) = \sqrt[3]{t} + \frac{1}{t}$.

Missing C : −1

(a) $x^2/2 \quad - 3x + C$ 3

(b) $3e^x \quad + 4\sin(x) + C$ 3

(c) $\frac{3}{4}t^{4/3} \quad + \ln|t| + C$ 3

(6) A particle is moving with acceleration

$$a(t) = \cos(t) + \sin(t)$$

where distance is measured in centimeters and time is measured in seconds. Its initial position is $s(0) = 0$ cm and its initial velocity is $v(0) = 5$ cm/sec. Find each of the following and state your answers with correct units.

(a) Find $v(t)$, the velocity of the particle as a function of time.

(b) Find $s(t)$, the position of the particle as a function of time.

~~cct~~

(a) $v(t) = \sin(t) - \cos(t) + C$ 1

$v(0) = -1 + C = 5$

$\therefore C = 6$ cm/sec 2

4

$v(t) = \sin(t) - \cos(t) + 6$ 1

(b) $s(t) = -\cos(t) - \sin(t) + 6t + C$ 1

$s(0) = 0 = -1 + C$

$\therefore C = 1$ 2

4

$s(t) = -\cos(t) - \sin(t) + 6t + 1$ 1

(a) $v(t) =$ $\sin(t) - \cos(t) + 6$

(b) $s(t) =$ $-\cos(t) - \sin(t) + 6t + 1$

⑨ (7) Find all horizontal and vertical asymptotes of the function

$$f(x) = \frac{x^2+4}{3x^2-6x+3}$$

and compute all limits that are needed to support your answer.

$$f(x) = \frac{x^2+4}{3(x^2-2x+1)} = \frac{x^2+4}{3(x-1)^2}$$

⑥

HA: $\lim_{x \to +\infty} \frac{x^2+4}{3(x^2-2x+1)} = \lim_{x\to\infty} \frac{x^2(1+4/x^2)}{x^2(3-6/x+3/x^2)}$ ②

$= \frac{1}{3}$ ①

$\lim_{x\to -\infty} \frac{x^2+4}{3(x^2-2x+1)} = \frac{1}{3}$ by a similar calculatn ②

$\therefore y = \frac{1}{3}$ is a HA ① (give eq'n)

⑥

③

VA: $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{x^2+4}{3(x-1)^2} = +\infty$

$\lim_{x\to 1^-} f(x) = \lim_{x \to 1^-} \frac{x^2+4}{3(x-1)^2} = +\infty$

at least one limit ②

$\therefore x = 1$ is VA ① (give eq'n)

Equation(s) of horizontal asymptote(s) $y = 1/3$

Equation(s) of vertical asymptote(s) $X = 1$

Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front page of the exam.

15

(8) (a) State the Mean Value Theorem. Use complete sentences.

5

Suppose f is differentiable on (a,b) and continuous on $[a,b]$. ① ①

There is a c between a and b so ①

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$ ② (formula)

(b) Does the Mean Value Theorem apply to the function $f(x) = |x-3|$ on the interval $[-4, 4]$? Explain why or why not.

5

No. f is not differentiable at $x=3$.

(c) Suppose that g is differentiable for all x and that $-2 \le g'(x) \le 3$ for all x. Furthermore, assume $g(0) = 1$. Use the Mean Value Theorem to show that $g(x) \le 1 + 3x$ for all $x \ge 0$.

5

Consider f on $[0,x]$. We have. ①

$$\frac{f(x)-f(0)}{x-0} = f'(c) \le 3$$ ②

$$\therefore \frac{f(x)-1}{x} \le 3$$ ①

$$\therefore f(x)-1 \le 3x$$

$$\therefore f(x) \le 1+3x$$ ①

(9) This problem concerns l'Hospital's Rule. In parts (b) and (c), be sure to explain clearly how you are applying l'Hospital's Rule to find the limits.

(a) State l'Hospital's Rule. Use complete sentences.

5

(1) (Suppose that $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ are both either 0 or $\pm\infty$,) and (suppose f, g are differentiable near $x=a$.) (1) with $g'(x) \neq 0$ near $x=a$ ~~but~~ except possibly at $x=a$ (1)

If $\lim_{x\to a} \frac{f'(x)}{g'(x)}$ exists, then (1)

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}. \quad (1)$$

5 (b) Find $\lim_{x\to 0^+} x \ln x$.

$$\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{1/x} \ (1) \qquad f(x) = \ln x \quad f'(x) = 1/x$$

$$(1)\ g(x) = 1/x \quad g'(x) = -1/x^2$$

(2)

$$\lim_{x\to 0^+} \frac{f'(x)}{g'(x)} = \lim_{x\to 0^+} \frac{1/x}{-1/x^2} = \lim_{x\to 0^+} (-x) = \boxed{0}$$

Hence, $\lim_{x\to 0^+} x \ln x = 0$ (1)

5 (c) Find $\lim_{x\to\infty} x^{1/x}$.

If $\lim_{x\to\infty} \ln(x^{1/x}) = L$ then $\lim_{x\to\infty} x^{1/x} = e^L$ (1)

$\ln(x^{1/x}) = \frac{1}{x} \ln x.$ Take $f(x) = \ln x \quad g(x) = x.$ (1)

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} g(x) = +\infty \quad (1)$$

$$\lim_{x\to\infty} \frac{f'(x)}{g'(x)} = \lim_{x\to\infty} \frac{1/x}{1} = 0 \implies \lim_{x\to+\infty} \ln(x^{1/x}) = 0 \quad (1)$$

$$\therefore \lim_{x\to\infty} x^{1/x} = e^0 = 1 \quad (1)$$

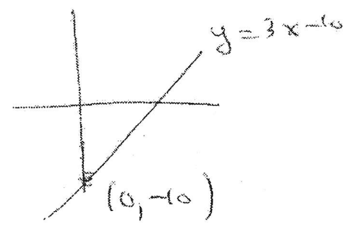
$\lim_{x\to 0^+} x\ln x =$ 0 $\qquad \lim_{x\to\infty} x^{1/x} =$ 1

(10) Find the point of the line $y = 3x - 10$ that is closest to the point $(0,0)$. Be sure to state clearly what function you choose to maximize or minimize and why.

(15)

$D = (3x-10)^2 + x^2$ (2) (square of distance) (3)

$-\infty < x < \infty$ (1)



$\frac{dD}{dx} = 2(3x-10)3 + 2x$

$= 18x - 60 + 2x$ (3)

$= 20x - 60$

(10) pts

$\therefore x = 3$ (2)

$y = 3(3) - 10 = -1$ (2)

$\therefore (x,y) = (3,-1)$

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(5) pts

Check abs min

$\frac{dD}{dx} = 20x - 60$
$= 20(x-3)$

Note $\frac{dD}{dx} < 0$ for $x < 3$

$\frac{dD}{dx} > 0$ for $x > 3$

By first derivative test for absolute minimum, ~~x=3~~ the absolute minimum of D occurs at $x = 3$

Point = $(3,-1)$