MA 113 Calculus I Fall 2017
Exam 3 Tuesday, 14 November 2017

Name: ______________________

Section: ______________________

Last 4 digits of student ID #: ________

This exam has 12 multiple choice questions (five points each) and 4 free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear earbuds during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box.*

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		60
13		10
14		10
15		10
16		10
Total		100

1. Suppose that $\dfrac{dy}{dt} = ky$, where k is a constant, and suppose that $y(0) = 4$ and $y(2) = 7$. Find $y(5)$.

   (A) $7 \cdot 7^5$

   (B) $7 \cdot 7^{5/2}$

   (C) $4 \cdot \left(\dfrac{7}{4}\right)^5$

   (D) $4 \cdot \left(\dfrac{7}{4}\right)^{5/2}$

   (E) None of the above

2. The half-life of a radioactive substance is 10 years. If a sample has a mass of 100 grams, how much of the sample remains after 27 years?

   (A) $100 \cdot e^{\ln(27)/10}$

   (B) $100 \cdot (1/2)^{27/10}$

   (C) $100 \cdot e^{\ln(1/2) \cdot 27}$

   (D) $100 \cdot e^{\ln(27/10)}$

   (E) $100 \cdot (1/2)^{10 \cdot 27}$

Record the correct answer to the following problems on the front page of this exam.

3. Suppose that $f'(x) = x(x-2)(x-4)$. Find the interval or intervals where f is increasing. (Read the problem carefully. The given function is $f'(x)$, not $f(x)$.)

   (A) $(0,2) \cup (4,\infty)$

   (B) $(-\infty,0) \cup (2,4)$

   (C) $(-\infty,0) \cup (4,\infty)$

   (D) $(2,\infty)$

   (E) $(0,4)$

4. Suppose that $f'(x) = x^3 + 6x^2 + 9x$. Find the interval or intervals where the graph of f is concave upward. (Read the problem carefully. The given function is $f'(x)$, not $f(x)$.)

   (A) $(-2,\infty)$

   (B) $(-3,\infty)$

   (C) $(-\infty,-3) \cup (-1,\infty)$

   (D) $(-3,-1)$

   (E) $(-\infty,-1)$

Record the correct answer to the following problems on the front page of this exam.

5. Find the critical points of $f(x) = \dfrac{x+1}{x^2+x+9}$.

   (A) $\{-2, 4\}$
   (B) $\{-4, 2\}$
   (C) $\{-1\}$
   (D) $\{-4, -2\}$
   (E) None of the above

6. Let $f(x) = \sqrt{x}$. Find a number c that satisfies the conclusions of the Mean Value Theorem over the interval $[9, 64]$.

   (A) $\dfrac{125}{4}$
   (B) $\dfrac{100}{4}$
   (C) $\dfrac{144}{4}$
   (D) $\dfrac{120}{4}$
   (E) $\dfrac{121}{4}$

Record the correct answer to the following problems on the front page of this exam.

7. You are given that $f'(x) = (x-1)(x-2)(x-3)^2(x-4)$. Find the values of x that give the local maximum and local minimum values of the function $f(x)$.

   (A) Local maximum value of f at $x = 2$ and local minimum values of f at $x = 1, 4$

   (B) Local maximum values of f at $x = 1, 4$ and local minimum value of f at $x = 2$

   (C) Local maximum value of f at $x = 3$ and local minimum values of f at $x = 1, 4$

   (D) Local maximum values of f at $x = 1, 3$ and local minimum values of f at $x = 2, 4$

   (E) Local maximum value of f at $x = 2, 4$ and local minimum values of f at $x = 1, 3$

8. Assume that $f''(x) = x^2(x-1)(x-3)$. Find the points of inflection of the function f.

   (A) $x = 0, 1, 3$

   (B) $x = 1$

   (C) $x = 3$

   (D) $x = 0, 3$

   (E) $x = 1, 3$

Record the correct answer to the following problems on the front page of this exam.

9. Assume that f is a differentiable function at all real numbers. Suppose that $f(1) = 7$ and that $-3 \leq f'(x) \leq 4$ at all real numbers. Use the Mean Value Theorem to determine the smallest possible value of $f(5)$.

   (A) -7

   (B) -6

   (C) -5

   (D) -4

   (E) -3

10. If x, y are positive numbers such that $x + y = 5$, then find the smallest possible value of $3x^2 + 4y^2$.

   (A) $42\,\frac{2}{7}$

   (B) $42\,\frac{3}{7}$

   (C) $42\,\frac{5}{7}$

   (D) $42\,\frac{6}{7}$

   (E) 43

Record the correct answer to the following problems on the front page of this exam.

11. Find the most general antiderivative of $\dfrac{3}{\sqrt{x}} + \sin(x) - 2x^{\frac{1}{4}}$.

(A) $-6\sqrt{x} + \cos(x) - \dfrac{4}{5}x^{\frac{5}{4}} + C$

(B) $6\sqrt{x} + \cos(x) - \dfrac{8}{5}x^{\frac{5}{4}} + C$

(C) $6\sqrt{x} - \cos(x) - \dfrac{8}{5}x^{\frac{5}{4}} + C$

(D) $6\sqrt{x} - \cos(x) - \dfrac{4}{5}x^{\frac{5}{4}} + C$

(E) $-6\sqrt{x} - \cos(x) - \dfrac{4}{5}x^{\frac{5}{4}} + C$

12. Which of the following definite integrals equals the following expression?

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{6}{n}\left(4 + \frac{-12i}{n} + \frac{36i^2}{n^2}\right)$$

(A) $\int_0^6 (4 - 12x + 6x^2)dx$

(B) $\int_0^6 (4 - 3x + 6x^2)dx$

(C) $\int_0^6 (4 - x + 2x^2)dx$

(D) $\int_0^6 (4 - 2x + x^2)dx$

(E) None of the above

Free Response Questions: Show your work!

13. A box with a square base and an open top is to have a volume of 81 m^3. Material for the base costs \$16 per square meter and material for the sides costs \$9 per square meter. Find the dimensions of the cheapest box.

Free Response Questions: Show your work!

14. You may use the following formulas in this problem.

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

(a) Compute the sum $\frac{3}{n}\sum_{i=1}^{n}(4 + \frac{2i}{n} + \frac{6i^2}{n^2})$.

(b) Compute

$$\lim_{n\to\infty}\left(3 + \frac{4}{n^2}\frac{n(n+1)}{2} + \frac{12}{n^3}\frac{n(n+1)(2n+1)}{6} - \frac{8}{n^4}\left(\frac{n(n+1)}{2}\right)^2\right).$$

Free Response Questions: Show your work!

15. Compute the following limits. Justify your work. No credit will be given for calculator computations.

(a) $\displaystyle \lim_{x \to -3} \frac{x^3 + 27}{x + 3}$

(b) $\displaystyle \lim_{x \to \infty} \frac{2x + \ln(x)}{x^2}$

(c) $\displaystyle \lim_{x \to 0} \frac{1 - \cos(x)}{x \sin(x)}$

Free Response Questions: Show your work!

16. Find $f(x)$ if $f''(x) = x - \dfrac{1}{x^2}$ and $f(1) = 3$, $f'(1) = \frac{7}{2}$.