**MA 113 — Calculus I** Spring 2013
**Exam 3** 09 April 2013

**Name:** ______________________________

**Section:** ____________

**Last 4 digits of student ID #:** ____________

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

**On the multiple choice problems:**

1. You must give your final answers in the multiple choice answer box on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the multiple choice answer box.

**On the free response problems:**

1. Clearly indicate your answer and the reasoning used to arrive at that answer (unsupported answers may not receive credit),
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

**Multiple Choice Answers**
**VERSION 1**

| Question | | | | | |
|---|---|---|---|---|---|
| 1 | A | B | C | **D** | E |
| 2 | A | B | **C** | D | E |
| 3 | A | **B** | C | D | E |
| 4 | A | B | C | **D** | E |
| 5 | **A** | B | C | D | E |
| 6 | **A** | B | C | D | E |
| 7 | A | **B** | C | D | E |
| 8 | **A** | B | C | D | E |
| 9 | **A** | B | C | D | E |
| 10 | A | B | C | **D** | E |

**Exam Scores**

| Question | Score | Total |
|---|---|---|
| MC | | 50 |
| 11 | | 10 |
| 12 | | 10 |
| 13 | | 10 |
| 14 | | 10 |
| 15 | | 10 |
| Total | | 100 |

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**Multiple Choice Answers**
**VERSION 2**

| Question | | | | | |
|---|---|---|---|---|---|
| 1 | A | **B** | C | D | E |
| 2 | A | B | C | D | **E** |
| 3 | A | B | C | **D** | E |
| 4 | A | **B** | C | D | E |
| 5 | A | B | **C** | D | E |
| 6 | A | **B** | C | D | E |
| 7 | **A** | B | C | D | E |
| 8 | A | B | C | D | **E** |
| 9 | A | B | **C** | D | E |
| 10 | A | B | **C** | D | E |

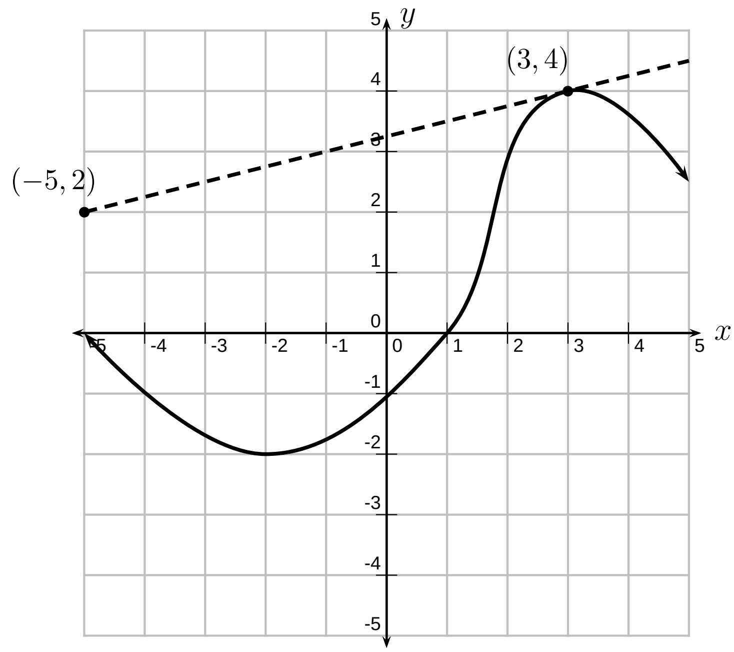
**Exam Scores**

| Question | Score | Total |
|---|---|---|
| MC | | 50 |
| 11 | | 10 |
| 12 | | 10 |
| 13 | | 10 |
| 14 | | 10 |
| 15 | | 10 |
| Total | | 100 |

**Record the correct answer to the following problems on the front page of the exam.**

THE FOLLOWING ARE THE QUESTIONS AND ANSWER CHOICES FOR VERSION 1 OF EXAM 3. VERSION 2 CONTAINED THE EXACT SAME QUESTIONS AND ANSWER CHOICES, ALTHOUGH THE ANSWER CHOICES WERE PERMUTED. THE ALTERNATE EXAM HAD DIFFERENT QUESTIONS.

**1.** The graph of $y = f(x)$ is shown below as the solid curve. The tangent line to $y = f(x)$ at the point $(3, 4)$ is also shown as the dashed line. Using linear approximation, estimate $f(3.02)$.




A. $f(3.02) \approx 3.980$

B. $f(3.02) \approx 3.995$

C. $f(3.02) \approx 4.000$

D. $f(3.02) \approx 4.005$

E. $f(3.02) \approx 4.020$

**2.** The general antiderivative of $g(x) = 12e^x - 5x^{-2}$ is

A. $12e^x + \frac{5}{3}x^{-3} + C$

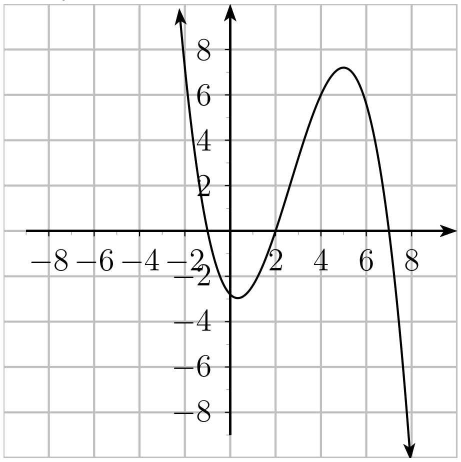
B. $e^{12x} + \frac{5}{3}x^{-3} + C$

C. $12e^x + 5x^{-1} + C$

D. $12e^{12x} + 5x^{-1} + C$

E. $e^{12x} + 10x^{-1} + C$

**3.** Let $f$ be a twice differentiable function with the graph of $\mathbf{y} = \mathbf{f}'(\mathbf{x})$ as shown below.




Which statement is NOT necessarily true?

A. $f(x)$ is decreasing on the interval $(-1, 2)$.

B. $f(x)$ is concave down on the interval $\left(\frac{1}{3}, 5\right)$.

C. $f(x)$ achieves a local maximum when $x = -1$.

D. $f(x)$ has an absolute maximum at $x = 0$ on the interval $\left[0, \frac{3}{2}\right]$.

E. $f''(x) = 0$ at $x = 5$.

**4.** Evaluate the sum

$$\sum_{k=1}^{43} (7k - 8)$$

A. 293

B. 860

C. 5977

D. 6278

E. 6614

**5.** A box is to have a square base, no top, and a volume of 108 cubic inches. What is the smallest possible total surface area?

A. 108 square inches

B. $6(108)^{2/3}$ square inches

C. 36 square inches

D. $\sqrt[3]{108}$ square inches

E. None of the above

**6.** Suppose you are estimating the value of $\sqrt{2}$ by finding the root of $x^2 - 2 = 0$ using Newton's method. If you use $x_1 = 1$, find the exact value of $x_3$.

A. $\dfrac{17}{12}$

B. $\dfrac{1}{144}$

C. $\dfrac{7}{5}$

D. $\dfrac{3}{2}$

E. None of the above.

**7.** Suppose that $g(x)$ is a continuous function with $g'(x) = \dfrac{x+4}{x-2}$. Which of the following is true?

A. $g(x)$ attains a local maximum at $x = 0$.

B. $g(x)$ attains a local maximum at $x = -4$.

C. $g(x)$ attains a local minimum at $x = 0$.

D. $g(x)$ attains a local minimum at $x = -4$.

E. None of the above.

**8.** Suppose $f(x)$ is a differentiable function such that the tangent line at $x = 3$ is given by $y = \dfrac{-1}{9}x + \dfrac{4}{3}$. Which of the following MUST be true?

A. According to the linearization of $f$ at $x = 3$, $f(3.01) \approx 0.9989$.

B. $f(3) = 0$

C. $f$ is concave down on an open interval containing $x = 3$.

D. The graph of $y = f(x)$ attains a maximum value on the interval $(-1, 4)$.

E. Applying Newton's Method to approximate the roots of $f(x) = 0$ with $x_1 = 3$, we have $x_2 = \dfrac{4}{3}$.

**9.** What is the absolute minimum value of the function below on the interval $[-3, 0]$?

$$f(t) = \sqrt[5]{(t+1)^2} + \frac{2t}{5}$$

A. $-\frac{2}{5}$

B. $\frac{1}{5}$

C. $-\frac{\sqrt[5]{4}}{5}$

D. $\sqrt[5]{4} - 1.2$

E. $\sqrt[5]{9} - 1.2$

**10.** Given the function $g(x) = x^2$, which value of $c$ satisfies the conclusion of the Mean Value Theorem on the interval $[-4, 5]$?

A. $\frac{-1}{\sqrt{2}}$

B. 0

C. 1

D. $\frac{1}{2}$

E. None of the above

**11.** Let $f(x) = e^{x^2}$. Use calculus to justify your answers.

(a) Give all critical points of $f$.

$$f'(x) = 2xe^{x^2}$$

**[Derivative 1 point]**

$f'$ **has domain** $(-\infty, \infty)$**, thus the only critical points occur when** $f'(x) = 0$**.** $2xe^{x^2} = 0$ **has one solution,** $x = 0$**.**

**[2 points for solving $f'(x) = 0$]**

Critical point: $x = 0$ **[Correction Solution 1 point]**

(b) Find the open intervals where $f$ is increasing or decreasing.

The sign of $2xe^{x^2}$ is:

| $-$ | $+$ |
|---|---|
| Test pt $x = -1$ | Test pt $x = 1$ |

$x = 0$

Increasing: $(0, \infty)$ **[1 point for correct answer]**

Decreasing: $(-\infty, 0)$ **[1 point for correct answer]**

(c) Find the intervals where $f$ is concave up or concave down.

$$f''(x) = 2e^{x^2} + 4x^2e^{x^2} = e^{x^2}(2 + 4x^2)$$

We have that $e^{x^2} > 0$ and $2 + 4x^2 > 0$ for all $x$, thus $f''(x) = e^{x^2}(2 + 4x^2) > 0$ for all $x$.

**[2 points for finding the sign of the second derivative]**

Concave up: $(-\infty, \infty)$ **[1 point for correct answer]**

Concave down: never **[1 point for correct answer]**

**12.** Suppose $h$ is a twice differentiable function with the following information given.

$h''(x) = \sin(x)$

$h'(\pi/2) = 0$

$h(\pi/2) = 0$

First find $h'(x)$. Then find $h(x)$.

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$h'(x)$ **is an antiderivative of** $h''(x)$**. Therefore,**

$$h'(x) = -\cos(x) + C$$

**2 points**

**From the information given,** $h'(\pi/2) = 0$**, yielding the equation**

$$0 = -\cos(\pi/2) + C$$
$$0 = C$$

**2 points**

$$h'(x) = -\cos(x)$$

**1 point**

---

$h(x)$ **is an antiderivative of** $h'(x)$**. Therefore,**

$$h(x) = -\sin(x) + C$$

**2 points**

**From the information given,** $h(\pi/2) = 0$**, yielding the equation**

$$0 = -\sin(\pi/2) + C$$
$$0 = -1 + C$$
$$1 = C$$

**2 points**

$$h(x) = -\sin(x) + 1$$

**1 point**

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**13.** (6 points) Evaluate the following limits. Explain your reasoning.

(a) $\lim_{x\to\infty} \frac{\sqrt{x}}{(\ln x)^2}$

**Both $\sqrt{x}$ and $(\ln x)^2$ approach $\infty$ as $x$ approaches $\infty$, so L'Hôpital's Rule applies to the quotient. [1 point for indeterminate form and invoking L'Hôpital's Rule.]**

$$\begin{aligned}\lim_{x\to\infty} \frac{\sqrt{x}}{(\ln x)^2} &= \lim_{x\to\infty} \frac{\frac{1}{2}x^{-1/2}}{\frac{2\ln x}{x}} \\ &= \lim_{x\to\infty} \frac{x}{4x^{1/2}\ln(x)} \\ &= \lim_{x\to\infty} \frac{x^{1/2}}{4\ln(x)}\end{aligned}$$

**[2 points for correct derivatives and simplification]**

**Both $\sqrt{x}$ and $4\ln x$ approach $\infty$ as $x$ approaches $\infty$, so L'Hôpital's Rule applies again. [1 point for invoking L'Hôpital's Rule again.]**

$$\begin{aligned}\lim_{x\to\infty} \frac{x^{1/2}}{4\ln(x)} &= \lim_{x\to\infty} \frac{\frac{1}{2}x^{-1/2}}{\frac{4}{x}} \\ &= \lim_{x\to\infty} \frac{x^{1/2}}{8} \\ &= \infty\end{aligned}$$

**[1 point for correct derivatives and simplification]**

**Therefore $\lim_{x\to\infty} \frac{\sqrt{x}}{(\ln x)^2}$ is infinite. [1 point for correct answer.]**

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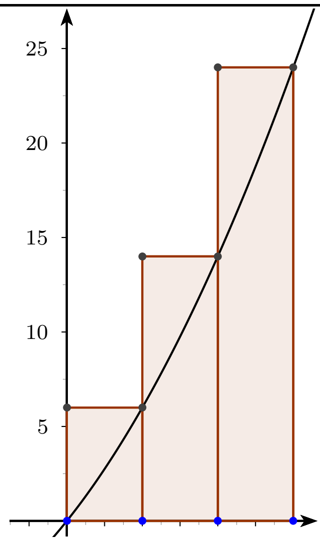
(b) (4 points) $\lim_{x\to\pi} \frac{\cos(\frac{x}{2})}{\sin(x)}$

**The functions $\cos(\frac{x}{2})$ and $\sin(x)$ both approach $0$ as $x$ approaches $\pi$, so L'Hôpital's Rule applies to the quotient. [1 point for indeterminate form and invoking L'Hôpital's Rule.]**

$$\begin{aligned}\lim_{x\to\pi} \frac{\cos(\frac{x}{2})}{\sin(x)} &= \lim_{x\to\pi} \frac{-\frac{1}{2}\sin(\frac{x}{2})}{\cos(x)} \\ &= \frac{-\frac{1}{2}\sin(\frac{\pi}{2})}{\cos(\pi)} = \frac{1}{2}\end{aligned}$$

**[2 points for correct derivatives, 1 point for correct answer]**

**14.** Let $f(x) = x^2 + 5x$. Subdivide the interval $[0, 3]$ into three equal sub-intervals and compute $R_3$, the value of the right-endpoint approximation to the area under the graph of $f$ on the interval $[0, 3]$.




[ Picture 2 points (or an indication that this is the sum of rectangles)]

$$\Delta x = \frac{3-0}{3} = 1$$

[Correct Width 2 points]

$$\begin{aligned}
\text{Height of Rectangle 1} &= f(1) = 6 \\
\text{Height of Rectangle 2} &= f(2) = 14 \\
\text{Height of Rectangle 3} &= f(3) = 24
\end{aligned}$$

[Correct rectangle heights 4 points]

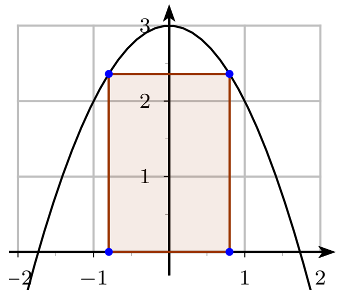
$$R_3 = 1 \cdot (6 + 14 + 24) = 44$$

[Correct Answer 2 points]

An Alternative Solution for question 14.

$$
\begin{aligned}
R_3 &= \Delta x \sum_{j=1}^{3} f(0 + k\Delta x)[\text{Correct Formula 3 points}] \\
&= \frac{3-0}{3} \sum_{j=1}^{3} f(k)[\text{Correct Width 2 points}] \\
&= 1 \cdot \sum_{j=1}^{3} (k^2 + 5k)[\text{Correct Height Formula 2 points}] \\
&= \sum_{j=1}^{3} k^2 + 5 \sum_{j=1}^{3} k \\
&= \frac{3 \cdot 4 \cdot 7}{6} + 5 \frac{3 \cdot 4}{2} \\
&= 44[\text{Correct Summation 3 points}]
\end{aligned}
$$

**15.** Find the dimensions of the rectangle of maximum area inscribed in the region bounded below by the $x$-axis and by the graph of $y = 3 - x^2$.



**The goal is to maximize the area of the rectangle whose base is $2x$ and whose height is $3 - x^2$.**

$$\textbf{Maximize } A(x) = 2x(3 - x^2)$$

**[3 points for the objective function]**

**The domain of the area function is $[0, \sqrt{3}]$**
**[2 points for the domain of the objective function]**

$$A'(x) = 6 - 6x^2$$

**$A'(x)$ is always defined and $6 - 6x^2 = 0$ for $x = \pm 1$. Only $x = 1$ is contained in the relevant interval.**

| $x$ | $A(x)$ |
|---|---|
| 0 | 0 |
| 1 | 4 |
| $\sqrt{3}$ | 0 |

**[2 points for testing endpoints and critical points]**

**By the Extreme Value Theorem, 4 is the maximum of the area function which occurs at $x = 1$. When $x = 1$, the dimensions of the rectangle are 2 units by 2 units.**
**[1 point for invoking EVT, 2 points for correct answer]**

**NOTE: The first derivative test and second derivative tests only guarantee that $x = 1$ is a local maximum.**