MA 113 Calculus I — Spring 2016

Exam 3 — Tuesday, April 12, 2016

Name: ____________________

Section: ____________________

Last 4 digits of student ID #: ________

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. Assume that $f'(x) = (x-2)(x-6)$. Which of the following statements about local maximum values and local minimum values of $f(x)$ is true?

   (A) A local maximum occurs at $x = 2$ and a local maximum occurs at $x = 6$.

   (B) A local maximum occurs at $x = 2$ and a local minimum occurs at $x = 6$.

   (C) A local minimum occurs at $x = 2$ and a local maximum occurs at $x = 6$.

   (D) A local minimum occurs at $x = 2$ and a local minimum occurs at $x = 6$.

   (E) There is not enough information given to find where local maximum and minimum values of $f(x)$ occur.

2. Assume that $f''(x) = (x-2)(x-6)$. Which of the following statements about the graph of $f(x)$ is true?

   (A) $f(x)$ is concave up on $(-\infty, 2)$ and $(6, \infty)$, and $f(x)$ is concave down on $(2, 6)$.

   (B) $f(x)$ is concave down on $(-\infty, 2)$ and $(6, \infty)$, and $f(x)$ is concave up on $(2, 6)$.

   (C) $f'(x)$ is decreasing on $(-\infty, 2)$ and $f'(x)$ is increasing on $(6, \infty)$.

   (D) $f(x)$ has no inflection points.

   (E) There is not enough information given to find the intervals of concavity on the graph of $f(x)$.

3. Suppose that $f(x)$ is a differentiable function with $6 \leq f'(x) \leq 10$ for all x in the interval $(1, 7)$. If $f(1) = 3$, then the Mean Value Theorem for $f(x)$ on the interval $[1, 7]$ implies that the largest possible value of $f(7)$ is

   (A) 53

   (B) 57

   (C) 60

   (D) 63

   (E) 67

Record the correct answer to the following problems on the front page of this exam.

4. Suppose that x and y are chosen such that $3x + y = 42$ and that xy is as large as possible. Then $y - x$ equals

   (A) 6

   (B) 8

   (C) 10

   (D) 12

   (E) 14

5. Assume that $f(x)$ is a function and that all derivatives of f are defined everywhere. Assume the following information.

$$f'(2) = 0, \quad f'(4) = 0,$$
$$f''(2) = 3, \quad f''(4) = -2.$$

   Which of the following statements is true?

   (A) f has a local maximum value at $x = 2$, a point of inflection at $x = 4$.

   (B) f has a local maximum value at $x = 2$, a local maximum value at $x = 4$.

   (C) f has a local minimum value at $x = 2$, a local minimum value at $x = 4$.

   (D) f has a local maximum value at $x = 2$, a local minimum value at $x = 4$.

   (E) f has a local minimum value at $x = 2$, a local maximum value at $x = 4$.

Record the correct answer to the following problems on the front page of this exam.

6. Find $\lim_{x\to\infty} \frac{\ln(x)}{\sqrt[3]{x}}$.

   (A) 0

   (B) 1

   (C) e

   (D) 3

   (E) ∞

7. Find $\int (3\sin(x) - \frac{2}{x^3})dx$.

   (A) $3\cos(x) + 6x^{-4} + C$

   (B) $3\cos(x) - \frac{1}{x^2} + C$

   (C) $3\cos(x) + \frac{1}{x^2} + C$

   (D) $-3\cos(x) - \frac{1}{x^2} + C$

   (E) $-3\cos(x) + \frac{1}{x^2} + C$

Record the correct answer to the following problems on the front page of this exam.

8. Let $f(x) = x^3 - x + 1$. Suppose we use Newton's Method to estimate a root of $f(x) = 0$. If the first estimate is $x = -1$, then find the second estimate that would be obtained by using Newton's Method.

   (A) $x = -2$

   (B) $x = \dfrac{-3}{2}$

   (C) $x = -1$

   (D) $x = \dfrac{-1}{2}$

   (E) $x = \dfrac{1}{2}$

9. Use L'Hôpital's Rule to find $\displaystyle\lim_{x \to a} \frac{x^2 - (a+7)x + 7a}{x^2 - a^2}$.

   (A) $\dfrac{1}{2}$

   (B) 1

   (C) $\dfrac{a-7}{2a}$

   (D) $\dfrac{a+7}{2a}$

   (E) ∞

10. $f'(x) = 3\sqrt{x}$ and $f(1) = 5$. Find $f(4)$.

   (A) 16

   (B) 17

   (C) 18

   (D) 19

   (E) 20

Free Response Questions: Show your work!

11. (a) Find the linearization $L(x)$ of $f(x) = \sqrt[3]{x}$ at $x = 8$.

(b) Use the method of linearization at $x = 8$ to approximate $\sqrt[3]{8.12}$.
(You must show your work. You will not receive credit for using a calculator computation.)

Free Response Questions: Show your work!

12. Find the point on the line $y = x + 2$ closest to the point $(9, 0)$.
    Be sure to explain why the point you found gives the minimum distance.
    (You must show your work to receive credit. Guessing the answer will not receive credit.)

Free Response Questions: Show your work!

13. $f''(x) = 6x + 4 - \dfrac{3}{x^2}$, $f'(1) = 6$, and $f(1) = 7$.

   (a) Find $f'(x)$.

   (b) Find $f(x)$.

Free Response Questions: Show your work!

14. Let $f(x) = x^4 - 18x^2 + 7$. Be sure to justify each of your answers below.

    (a) Find the intervals where $f(x)$ is increasing and the intervals where $f(x)$ is decreasing.

    (b) Find the intervals where $f(x)$ is concave up and the intervals where $f(x)$ is concave down.

    (c) Find the points that give local maximum values of $f(x)$, the points that give local minimum values of $f(x)$, and the points of inflection of $f(x)$.

15. Recall the following formulas.

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

Recall that R_N denotes the N^{th} right-endpoint approximation of the area under the graph of $f(x)$ and that

$$R_N = \Delta x \sum_{j=1}^{N} f(a + j\Delta x), \text{ where } \Delta x = \frac{b-a}{N}.$$

(a) Find R_4 for the function $f(x) = 4x^2 + 2x$ on the interval $[1, 3]$.

(b) Find a formula for R_N for any N for the function $f(x) = 4x^2 + 2x$ on the interval $[1, 3]$.

(c) Find $\lim\limits_{N \to \infty} R_N$.