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Assignment Exam03 due 04/14/2020 at 07:00pm EDT

Find the number c that satisfies the conclusion of the Mean Value Theorem for the function $f(x) = 2\sqrt{x}$ on the interval $[0,25]$.

- A. 0
- B. $\frac{25}{4}$
- C. $\frac{1}{5}$
- D. 5
- E. None of the above

Correct Answers:

- B

Find the value of the limit

$$\lim_{x\to 0} \frac{\sin 4x - 4x}{x^3}.$$

- A. $-\frac{2}{3}$
- B. $-\frac{16}{3}$
- C. $-\frac{32}{3}$
- D. $-\frac{4}{3}$
- E. None of the above

Correct Answers:

- C

Find two positive numbers x and y whose sum is 7 so that $x^2y - 8x$ is a maximum.

- A. $\frac{3}{2}, \frac{11}{2}$
- B. $5, 2$
- C. $\frac{7}{2}, \frac{7}{2}$
- D. $4, 3$
- E. $6, 1$

Correct Answers:

- D

Find the intervals where $f(x) = \dfrac{\ln(2x)}{x}$ is increasing and where it is decreasing.

- A. increasing on $(20, \infty)$, decreasing on $(0, 20)$.
- B. increasing on $\left(0, \dfrac{e}{2}\right)$, decreasing on $\left(\dfrac{e}{2}, \infty\right)$
- C. increasing on $(2e, \infty)$, decreasing on $(0, 2e)$
- D. increasing on $\left(0, \dfrac{1}{5}\right)$, decreasing on $\left(\dfrac{1}{5}, \infty\right)$
- E. None of the above

Correct Answers:

- `B`

Find the critical number(s) of the function $f(x) = e^{x^2 - 10x}$.

- A. 5
- B. $\sqrt{10}$ and $-\sqrt{10}$
- C. 5 and -5
- D. $\sqrt{10}$
- E. The function has no critical numbers.

Correct Answers:

- `A`

Find the local maxima and local minima, if any, of the function $f(x) = 2x^3 - 3x^2 - 36x - 5$.

- A. The local maximum is $f(3) = 86$ and the local minimum is $f(-2) = -39$.
- B. The local maximum is $f(-3) = 22$ and the local minimum is $f(2) = -73$.
- C. The local maximum is $f(2) = 73$ and the local minimum is $f(-3) = -22$.
- D. The local maximum is $f(-2) = 39$ and the local minimum is $f(3) = -86$.
- E. None of the above.

Correct Answers:

- `D`

Suppose that $f'(x) = x^2(x+2)(x-2)(x-4)$. Find the open interval or open intervals where f is decreasing. (Read the problem carefully. The given function is $f'(x)$, not $f(x)$.)

- A. $(-2, 2) \cup (4, \infty)$
- B. $(-\infty, -2) \cup (2, \infty)$
- C. $(-2, 2) \cup (4, \infty)$
- D. $(2, 4)$
- E. $(-\infty, -2) \cup (2, 4)$

Correct Answers:

- `E`

You are given that $f'(x) = x^2(x+2)(x-2)(x-4)$. Find the values of x that give the local maximum and local minimum values of the function $f(x)$. (Read the problem carefully. The given function is $f'(x)$, not $f(x)$.)

- A. Local maximum value of f at $x = 0$ and local minimum values of f at $x = -2, 4$.
- B. Local maximum values of f at $x = -2, 4$ and local minimum value of f at $x = 0$.
- C. Local maximum value of f at $x = 2$ and local minimum values of f at $x = -2, 4$.
- D. Local maximum values of f at $x = -2, 2$ and local minimum values of f at $x = 0, 4$.
- E. Local maximum values of f at $x = 0, 4$ and local minimum values of f at $x = -2, 2$.

Correct Answers:

- C

Assume that $f''(x) = x^2(x-2)(x-4)$. Find the points of inflection of the function f. (Read the problem carefully. The given function is $f''(x)$, not $f(x)$.)

- A. $x = 4$
- B. $x = 2, 4$
- C. $x = 2$
- D. $x = 0, 2, 4$
- E. $x = 0, 4$

Correct Answers:

- B

If $f''(x) = 18x + 36x^2$, find $f(x)$.

- A. $f(x) = 2x^3 + x^4 + Cx + D$
- B. $f(x) = 3x^3 + 3x^4 + Cx + D$
- C. $f(x) = x^3 + 4x^4 + Cx + D$
- D. $f(x) = 4x^3 + 8x^4 + Cx + D$
- E. $f(x) = 6x^3 + 4x^4 + Cx + D$

Correct Answers:

- B

Use the Fundamental Theorem of Calculus to find the derivative of the function:

$$g(x) = \int_5^{x^2} t^5 \sin(t) dt.$$

- A. $g'(x) = 2x^{11} \sin(x^2)$
- B. $g'(x) = 10x^5 \sin(x^2)$
- C. $g'(x) = 5x^8 \sin(x^2)$
- D. $g'(x) = 5x^8 \cos(x^2)$
- E. $g'(x) = x^{10} \sin(x^2)$

Correct Answers:

- A

An object travels with a velocity of 10 m/s for $0 \leq t \leq 3$ seconds and a velocity of 15 m/s for $3 < t \leq 5$ seconds. How far did it travel?

- A. 60 meters
- B. 70 meters
- C. 50 meters
- D. 65 meters
- E. None of the above.

Correct Answers:

- `A`

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13. (10 points) `Library/Valdosta/APEX_Calculus/3.3/APEX_3.3_23.pg`

NOTE: When using interval notation in WeBWorK, remember that:
You use 'INF' for ∞ and '-INF' for $-\infty$.
And use 'U' for the union symbol.
Enter **DNE** if an answer does not exist.

$$f(x) = x^3 - 3x$$

a) Find the critical numbers of f. ______ (Separate multiple answers by commas.)

b) Determine the intervals on which f is increasing and decreasing.
f is increasing on: ______
f is decreasing on: ______

c) Use the First Derivative Test to determine whether each critical point is a relative maximum, minimum, or neither.
Relative maxima occur at $x =$ ______ (Separate multiple answers by commas.)
Relative minima occur at $x =$ ______ (Separate multiple answers by commas.)

Solution: *(Instructor solution preview: show the student solution after due date.)*

Solution:

$f'(x) = 3x^2 - 3$. Set equal to zero and solve.

$$\begin{aligned} 3(x^2 - 1) &= 0 \\ x^2 &= 1 \\ x &= \pm 1 \end{aligned}$$

There are two critical numbers, $x = -1, 1$.

Use the first derivative test, choosing sample points in each interval.

Interval	Sign of f' at sample	Conclusion
$(-\infty, -1)$	positive	increasing
$(-1, 1)$	negative	decreasing
$(1, \infty)$	positive	increasing

There is a relative maximum at $x = -1$ and a relative minimum at $x = 1$

Correct Answers:

- `-1, 1`
- `(-infinity,-1) U (1,infinity)`
- `(-1,1)`

- -1
- 1

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14. **(5 points)** `Library/Valdosta/APEX_Calculus/6.7/APEX_6.7_17.pg`

Evaluate the limit, using L'Hôpital's Rule.

Enter **INF** for ∞, **-INF** for $-\infty$, or **DNE** if the limit does not exist, but is neither ∞ nor $-\infty$.

$\lim_{x\to 0} \frac{2e^x - 2x - 2}{15x^2} =$ ____

Solution: *(Instructor solution preview: show the student solution after due date.)*

$$\lim_{x\to 0} \frac{2e^x - 2x - 2}{15x^2} = \lim_{x\to 0} \frac{2e^x - 2}{30x} = \lim_{x\to 0} \frac{2e^x}{30} = \frac{2}{30} = \frac{1}{15}.$$

Correct Answers:

- `2/30`

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15. **(10 points)** `Library/Union/setDervConcavity/4-3-52.pg`

Let $f(x) = -x^4 - 4x^3 + 2x + 4$. Find the open intervals on which f is concave up (down). Then determine the x-coordinates of all inflection points of f.

1. f is concave up on the intervals ____
2. f is concave down on the intervals ____
3. The inflection points occur at $x =$ ____

Notes: In the first two, your answer should either be a single interval, such as (0,1), a comma separated list of intervals, such as (-inf, 2), (3,4), or the word "none".

In the last one, your answer should be a comma separated list of x values or the word "none".

Correct Answers:

- `(-2,0)`
- `(-infinity,-2), (0,infinity)`
- `0, -2`

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16. **(5 points)** `Library/UCSB/Stewart5_4_10/Stewart5_4_10_3.pg`

Find the most general antiderivative of $f(x) = -4 - 2x^3 - 8x^5 - 6x^7$.

Note: Any arbitrary constants used must be an upper-case "C".

$F(x) =$ ____

Correct Answers:

- `-4*x+-2*x^4/4+-8*x^6/6+-6*x^8/8+C`

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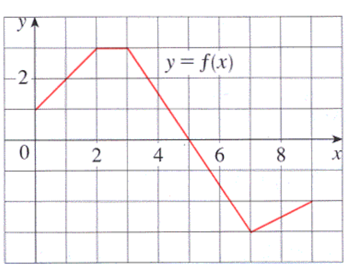
17. **(5 points)** `Library/UCSB/Stewart5_5_2/Stewart5_5_2_33/Stewart5_5_2_33.pg`

Consider the graph of the function $f(x)$:



Evaluate the following integrals by interpreting them in terms of areas:

(a) $\int_0^2 f(x)\,dx =$ ____

(b) $\int_0^5 f(x)\,dx =$ ____________

(c) $\int_5^7 f(x)\,dx =$ ____________

(d) $\int_0^9 f(x)\,dx =$ ____________

Correct Answers:

- 4
- 10
- -3
- 2

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18. **(5 points)** `Library/Wiley/setAnton_Section_5.6/Anton_5_6_Q60.pg`

Use the Fundamental Theorem of Calculus to find the derivative.

$\frac{d}{dx}\int_1^x \frac{dt}{8+\sqrt{t}} =$ _________

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

Using Part 2 of the Fundemental Theorem of Calculus

$$\frac{d}{dx}\int_1^x \frac{dt}{8+\sqrt{t}} = \frac{1}{8+\sqrt{x}}$$

Correct Answers:

- 1/[8+sqrt(x)]