Exam 3
Solutions

Multiple Choice Questions

1. If $\int_0^9 f(x)\,dx = 6$ and $\int_0^9 g(x)\,dx = 5$, find $\int_0^9 (5f(x) - 7g(x) + 2)\,dx$

   A. -5

   B. 13

   C. 65

   D. 67

   E. 83

2. Find the general antiderivative of $f(x) = 1/x + \sin(x) + 2\cos(x)$ on $(0, \infty)$.

   A. $-1/x^2 - \cos(x) + 2\sin(x) + C$

   B. $1/x^2 + \cos(x) + 2\sin(x) + C$

   C. $\ln(x) + \cos(x) + 2\sin(x) + C$

   D. $\ln(x) - \cos(x) + 2\sin(x) + C$

   E. $\ln(x) + \cos(x) - 2\sin(x) + C$

3. Find the largest area of a rectangle if its perimeter is 60 meters.

   A. 15 square meters

   B. 32 square meters

   C. 50 square meters

   D. 225 square meters

   E. 900 square meters

4. Suppose f is a differentiable function, $f(2) = 3$ and $f'(x) \leq 6$ for $2 \leq x \leq 4$, how large can $f(4)$ possibly be?

   A. 6

   B. 9

   C. 12

   D. 13

   E. 15

5. Find all of the critical numbers for the function $g(x) = x^3 - 2x^2 - 4x + 144$.

   A. $x = 0$ only

   B. $x = 2$ only

   C. $x = \pm 12$

   D. $x = -\frac{2}{3}$ and $x = 2$

   E. $x = 3$ and $x = 2$

6. Find the value of the limit
$$\lim_{x \to 0} \frac{3\sin(4x) - 12x}{x^3}.$$

   A. -2

   B. -4

   C. -8

   D. -16

   E. -32

7. If $\int_0^3 f(x)dx = 13$ and $\int_0^2 f(x)dx = 7$, find $\int_2^3 f(x)dx$.

   A. -6

   B. 6

   C. 7

   D. 13

   E. 20

8. Find $f(x)$ if $f'(x) = 3x^2 - 2\sin(x)$ and $f(0) = 5$.

   A. $f(x) = x^3 + 2\cos(x)$

   B. $f(x) = 6x - 2\sin(x) + 5$

   C. $f(x) = x^3 + 2\cos(x) + 3$

   D. $f(x) = x^3 + 2\cos(x) - 5$

   E. $f(x) = x^3 - 2\cos(x) + 7$

9. Where does the function $f(x) = x^3 - 9x^2$ have a point of inflection?

    A. $x = -4$

    B. $x = 0$

    C. $x = 1$

    D. $x = 2$

    E. $x = 3$

10. An athlete runs with velocity 24 km/h for 10 minutes, 18 km/h for 5 minutes , and 30 km/h for 5 minutes. Compute the total distance traveled.

    A. 5 km

    B. 6 km

    C. 7 km

    D. 8 km

    E. 9 km

11. Find the absolute maximum value of $f(x) = x^3 - 6x^2 + 9x - 5$ on the interval $[0, 5]$.

A. 10

B. 14

C. 15

D. 20

E. 48

12. Find $\int_0^4 f(x)\,dx$ if

$$f(x) = \begin{cases} 2 & x < 2 \\ -2x + 1 & x \geq 2 \end{cases}$$

A. -12

B. -6

C. 0

D. 8

E. 22

13. You are given that $f'(x) = x^2(x+2)(x-2)(x-4)$. Find the values of x that give the local maximum and local minimum values of the function $f(x)$. (Read the problem carefully. The given function is $f'(x)$, not $f(x)$.)

    A. Local maximum value of $f(x)$ at $x = 0$ and local minimum values of $f(x)$ at $x = -2, 4$.

    B. Local maximum value of $f(x)$ at $x = 2$ and local minimum values of $f(x)$ at $x = -2, 4$.

    C. Local maximum values of $f(x)$ at $x = -2, 4$ and local minimum value of $f(x)$ at $x = 0$.

    D. Local maximum values of $f(x)$ at $x = -2, 2$ and local minimum values of $f(x)$ at $x = 0, 4$.

    E. Local maximum values of $f(x)$ at $x = 0, 4$ and local minimum values of $f(x)$ at $x = -2, 2$.

14. Assume that $g''(x) = x^2(x-2)(x-4)$. Find the points of inflection of the function $g(x)$. (Read the problem carefully. The given function is $g''(x)$, not $g(x)$.)

    A. $x = 2$

    B. $x = 4$

    C. $x = 0, 4$

    D. $x = 2, 4$

    E. $x = 0, 2, 4$

Free Response Questions
Show all of your work

15. (a) Find the following limit:

$$\lim_{x\to 0} \frac{1-e^x}{\ln(x+1)}$$

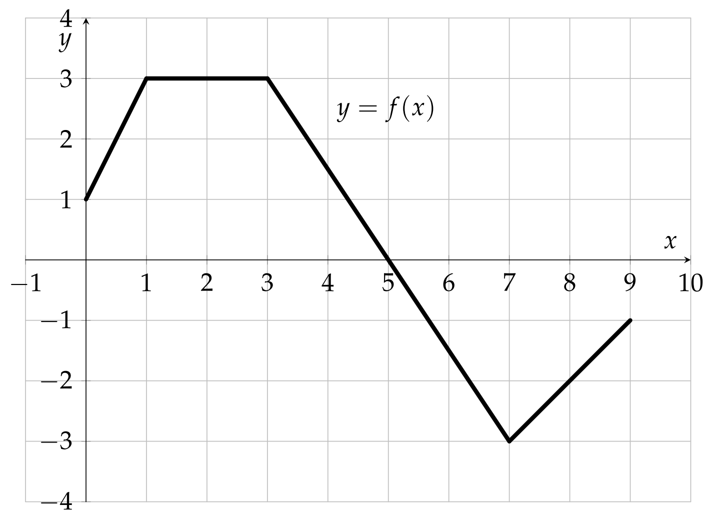
Solution: Using l'Hôpital's Rule, we have

$$\lim_{x\to 0} \frac{1-e^x}{\ln(x+1)} = \lim_{x\to 0} \frac{-e^x}{1/x+1} = -1$$

(b) Find the value of A for which we can use l'Hôpital's rule to evaluate the limit $\lim_{x\to 2} \frac{x^2+Ax-2}{x-2}$ and find the value of the limit.

Solution: We need $x^2 + Ax - 2$ to be zero when $x = 2$ then we can use l'Hôpital's Rule. At $x = 2$ we need $2^2 + 2A - 2 = 0$ or $A = -1$. Thus,

$$\lim_{x\to 2} \frac{x^2-x-2}{x-2} = \lim_{x\to 2} 2x - 11 = 3$$



16. A graph of $f(x)$ is shown above. Using the geometry of the graph, evaluate the definite integrals. The grid lines in the above graph are one unit apart.

(a) $\int_0^2 f(x)\,dx$

Solution: $\int_0^2 f(x)\,dx = 5$

(b) $\int_0^5 f(x)\,dx$

Solution: $\int_0^5 f(x)\,dx = 11$

(c) $\int_5^7 f(x)\,dx$

Solution: $\int_5^7 f(x)\,dx = -3$

(d) $\int_3^7 f(x)\,dx$

Solution: $\int_3^7 f(x)\,dx = 0$

(e) $\int_0^9 f(x)\,dx$

Solution: $\int_0^9 f(x)\,dx = 4$

17. Let $f(x) = x^4 - 32x^2 + 7$. Be sure to justify each of your answers below.

(a) Find the intervals where $f(x)$ is increasing and the intervals where $f(x)$ is decreasing.

Solution: $f'(x) = 4x^3 - 64x$. Setting $f'(x) = 0$, we get three critical points: $x = 0, x = 4, x = -4$. Checking the sign of the derivative, $f'(x) < 0$ for $x < -4$ and for $0 < x < 4$ so the function is decreasing on $(-\infty, -4)$ and on $(0, 4)$. It is increasing on $(-4, 0)$ and on $(4, \infty)$.

(b) Find the intervals where $f(x)$ is concave up and the intervals where $f(x)$ is concave down.

Solution: $f''(x) = 12x^2 - 64$. There are possible inflection points at $x = \pm 4/\sqrt{3}$. Checking the signs of the second derivative, we find that $f''(x) > 0$ on $(-\infty, -4/\sqrt{3})$ and on $(4/\sqrt{3}, \infty)$ so it is concave up there. It is concave down on the interval $(-4/\sqrt{3}, 4/\sqrt{3})$.

(c) Find the points that give local maximum values of $f(x)$, the points that give local minimum values of $f(x)$, and the points of inflection of $f(x)$.

Solution: $f(x)$ has a local maximum of 7 at $x = 0$ and local minimum value of -249 at $x = -4$ and $x = 4$. The x-coordinates of the points of inflection are $\pm 4/\sqrt{3}$.