MA 113 — Calculus I Fall 2010
Exam 3 November 16, 2010

Answer all of the questions 1 - 7 and **two** of the questions 8 - 10. Please indicate which problem is not to be graded by crossing through its number in the table below.

Additional sheets are available if necessary. No books or notes may be used. Please, turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities. Please:

1. clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Name: ____________________

Section: ________

Last four digits of student identification number: ________

Question	Score	Total
1		10
2		9
3		9
4		8
5		12
6		12
7		8
8		14
9		14
10		14
Free	4	4
		100

(1) Consider the function $g(t) = t^4 + 8t^3 + 18t^2 + 5$ on the interval $(-\infty, \infty)$.

(a) Find the critical number(s) of $g(t)$.

(b) Find the interval(s) of increase and decrease for $g(t)$.

(c) Find all local extreme values of $g(t)$. For each extremum, give both coordinates and specify whether it is a local minimum or a local maximum.

(a) Critical number(s): ____________________________

(b) Interval(s) of increase and decrease: ____________________

(c) Local maxima (both coordinates): ____________________

Local minima (both coordinates): ____________________

(2) Let $f(x) = (x+4)^{1/3}$.

(a) Find the linear approximation $L(x)$ to $f(x)$ at $x = 4$.

(b) Use the linear approximation you found in part (a) to estimate $(8.25)^{1/3}$. **Present your answer as a rational number.**

(a) $L(x) =$ ____________________________

(b) $(8.25)^{1/3} \approx$ ____________________________

(3) Consider the function

$$h(x) = x + \frac{1}{x}.$$

Find the absolute minimum and maximum values of h on the interval $\left[\frac{1}{2}, 2\right]$. Be sure to specify all the values of x where the absolute minimum and maximum are achieved.

The absolute maximum ______________ is taken on at ________________________.

The absolute minimum ______________ is taken on at ________________________.

(4) Consider the function f defined on the interval $(0, \pi)$ by $f(\theta) = \frac{\theta^2}{4} + \sin\theta$.

(a) Find the interval(s) where the graph of f is concave up or concave down; show your work.

(b) Find the point(s) of inflection of the graph of f; show your work.

(a) Interval(s) where the graph is concave up: ____________________

Interval(s) where the graph is concave down: ____________________

(b) Point(s) of inflection: ____________________________________

(5) Find the general antiderivative each of the following functions.

(a) $f(x) = \frac{1}{4}x^3 + 6x^2 + 1$.

(b) $g(t) = \csc^2 t - \sin t$.

(c) $h(v) = \frac{1}{v^2} + e^v$.

(a) ______________________________

(b) ______________________________

(c) ______________________________

(6) Consider the function

$$f(x) = \frac{\sqrt{x^6+2}+1}{x^3-8}.$$

Find all vertical and horizontal asymptotes of the curve $y = f(x)$. Be sure to compute all limits that are needed to justify your answer.

Horizontal asymptotes: ______________________

Vertical asymptotes: ______________________

(7) Evaluate the following limits using l'Hopital's Rule:

(a) $\lim\limits_{x\to\pi} \dfrac{\cos x + 1}{x^2 - \pi^2}$.

(b) $\lim\limits_{x\to\infty} x\left(\arctan x - \dfrac{\pi}{2}\right)$.

(a) $\lim\limits_{x\to\pi} \dfrac{\cos x + 1}{x^2 - \pi^2} =$ ____________________

(b) $\lim\limits_{x\to\infty} x\left(\arctan x - \dfrac{\pi}{2}\right) =$ ________________

Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front of the exam.

(8) (a) State Fermat's Theorem. Use complete sentences.

Parts (b) and (c): State whether the given functions have any local extreme values on the given intervals. If so, use an appropriate test to determine whether the extrema are local minima or local maxima. If not, explain how you can be sure that there are no extreme values.

(b) $f(x) = 0.99x + \cos x$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

(c) $g(x) = 1.01x + \cos x$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

(9) (a) State the Mean Value Theorem. Use complete sentences.

(b) Let $f(x) = (x-4)^2$. Without finding c, use the Mean Value Theorem to show that there is a number c in the interval $(3,5)$ such that $f'(c) = 0$.

(c) Let $g(x) = (x-4)^{-2}$. Show that there is no value of c in the interval $(3,5)$ such that $g(5) - g(3) = g'(c)(5-3)$, and explain why this does not contradict the Mean Value Theorem.

(10) Find the area of the largest rectangle that can be inscribed in the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$. (You may assume that the rectangle with largest area has sides which are parallel to the axes.)

Area = ____