MA 113 Calculus I Fall 2014
Exam 4 Wednesday, 17 December 2014

Name: ______________________

Section: ______________________

Last 4 digits of student ID #: ________

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

B,A,D,C,E, E,D,B,A,E

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. Find the equation of the line tangent to the function $f(x) = 4x^4$ and parallel to the line given by the equation $2x - y = 9$.

   (A) $y = 2x - 1$
   (B) $y = 2x - \frac{3}{4}$
   (C) $y = -2x + 1$
   (D) $y = 2x + \frac{1}{2}$
   (E) None of the above

2. Find the linearization $L(x)$ of the function $f(x) = 1 + \cos(x)$ at $a = \frac{\pi}{4}$.

   (A) $L(x) = (1 + \frac{\sqrt{2}}{2}) - \frac{\sqrt{2}}{2}(x - \frac{\pi}{4})$
   (B) $L(x) = (1 + \frac{\sqrt{2}}{2}) + \frac{\sqrt{2}}{2}(x - \frac{\pi}{4})$
   (C) $L(x) = (1 + \frac{\sqrt{2}}{2}) - \frac{\sqrt{2}}{2}(x + \frac{\pi}{4})$
   (D) $L(x) = (1 + \frac{\sqrt{2}}{2}) + \frac{\sqrt{2}}{2}(x + \frac{\pi}{4})$
   (E) $L(x) = (1 - \frac{\sqrt{2}}{2}) - \frac{\sqrt{2}}{2}(x - \frac{\pi}{4})$

3. Suppose $f(x) = x^2[g(x)]^3$, $g(4) = 2$, and $g'(4) = 3$. Find $f'(4)$.

   (A) 208
   (B) 310
   (C) 425
   (D) 640
   (E) 696

Record the correct answer to the following problems on the front page of this exam.

4. Find $\lim_{x \to 0} \dfrac{\tan(3x)}{5x}$.

   (A) 0

   (B) $\dfrac{3}{25}$

   (C) $\dfrac{3}{5}$

   (D) $\dfrac{5}{3}$

   (E) $\dfrac{9}{25}$

5. Suppose that a function f is defined by

$$f(x) = \begin{cases} \sqrt{x}, & 0 < x < 1 \\ c, & x = 1 \\ x^2 + 1, & x > 1 \end{cases}$$

   For what choice of c is f continuous at $x = 1$?

   (A) 1

   (B) 2

   (C) 3

   (D) 4

   (E) None of the above

Record the correct answer to the following problems on the front page of this exam.

6. Evaluate $\int_{-2}^{1} 3 + 2|x|\,dx$.

   (A) 10
   (B) 11
   (C) 12
   (D) 13
   (E) 14

7. Suppose $G(x) = \int_{1}^{x^3} te^t\,dt$. Find $G'(2)$.

   (A) $52e^4$
   (B) $96e^4$
   (C) $52e^8$
   (D) $96e^8$
   (E) None of the above

Record the correct answer to the following problems on the front page of this exam.

8. The doubling time of a population of bacteria is 10 hours. Suppose the initial population is 100. What is the population after 25 hours?

   (A) $200\sqrt{2}$

   (B) $400\sqrt{2}$

   (C) 450

   (D) 600

   (E) 650

9. Find $\displaystyle\int \frac{[\ln(x)]^2}{x}\,dx$

   (A) $\frac{1}{3}[\ln(x)]^3 + C$

   (B) $x[\ln(x)]^3 + C$

   (C) $\dfrac{[\ln(x)]^3}{x} + C$

   (D) $\dfrac{[\ln(x)]^3}{x^2} + C$

   (E) $\dfrac{2}{3}\dfrac{[\ln(x)]^3}{x^2} + C$

10. A right cylinder with radius 2 and height 20, both measured in inches, is being filled with water. The water pours in at the rate of 10 cubic inches per second. Find the rate at which the level of the water is rising in the tank.

    (A) $\dfrac{1}{5}$

    (B) 1

    (C) 5

    (D) $\dfrac{5}{\pi}$

    (E) $\dfrac{5}{2\pi}$

Free Response Questions: Show your work!

11. Consider the functions $f(x) = \sqrt{x}$ and $g(x) = x^2$.

    (a) Sketch and label the graphs of f and g.

    (b) Find the intersection points of f and g.
    $\sqrt{x} = x^2$, $x = x^4$, $x(x^3 - 1) = 0$, $x = 0, 1$. The intersection points are $(0, 0)$ and $(1, 1)$.

    (c) Find the area of the region bounded by $f(x)$ and $g(x)$ between the vertical lines $x = 0$ and $x = 2$.
    The area is $\int_0^1 (\sqrt{x} - x^2)dx + \int_1^2 (x^2 - \sqrt{x})dx = (\frac{2}{3}x^{3/2} - \frac{1}{3}x^3)|_0^1 + (\frac{1}{3}x^3 - \frac{2}{3}x^{3/2})|_1^2$
    $= (\frac{2}{3} - \frac{1}{3}) + (\frac{8}{3} - \frac{2 \cdot 2\sqrt{2}}{3}) - (\frac{1}{3} - \frac{2}{3}) = \frac{10 - 4\sqrt{2}}{3}$.

Free Response Questions: Show your work!

12. The base of a rectangle is on the x-axis and the other two corners are above the x-axis lying on the curve given by $y = 6 - x^2$.

    (a) Sketch the curve $y = 6 - x^2$ and this rectangle.

    (b) Express the area of the rectangle as a function of a single variable.
    Assume that (x, y) and $(-x, y)$ are the coordinates of the two corners lying above the x-axis. The area $A(x)$ is given by $2xy = 2x(6 - x^2) = 12x - 2x^3$.

    (c) Find the dimensions and area of the largest such rectangle. Justify your answer.
    To find the maximum on the interval $[-\sqrt{6}, \sqrt{6}]$, we set $A'(x) = 12 - 6x^2 = 0$. This gives $x = \pm\sqrt{2}$, and so $y = 4$. This is an absolute maximum on this interval. Thus the dimensions of the largest rectangle are $2\sqrt{2}$ by 4, and the maximal area is $2\sqrt{2} \cdot 4 = 8\sqrt{2}$.

Free Response Questions: Show your work!

13. Evaluate

(a) $\displaystyle\int_0^1 \frac{e^{2x}}{1+e^{2x}}\,dx$

Let $u = 1 + e^{2x}$. Then $du = 2e^{2x}dx$. The integral becomes $\int_2^{1+e^2} \frac{1}{2}\frac{du}{u} = \frac{1}{2}\ln(u)|_2^{1+e^2} = \frac{1}{2}(\ln(1+e^2) - \ln(2)) = \frac{1}{2}\ln(\frac{1+e^2}{2})$.

(b) $\displaystyle\int x\sqrt{x+1}\,dx$

Let $u = x + 1$. Then $du = dx$. The integral becomes $\int (u-1)u^{1/2}du = \int (u^{3/2} - u^{1/2})du = \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C = \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C$.

Free Response Questions: Show your work!

14. Assume that the derivative of a function $f(x)$ satisfies $f'(x) = xe^{-x}$.

   (a) Find the intervals over which f is increasing, the intervals where f is decreasing, and find all the local minima and maxima of f.

   If $f'(x) = 0$, then $x = 0$. Since $e^{-x} > 0$ for all x, it follows that $f'(x) < 0$ when $x < 0$, and $f'(x) > 0$ when $x > 0$. Thus f is decreasing for $x < 0$ and f is increasing for $x > 0$. The value $x = 0$ is a local minimum by the first derivative test.

   (b) Find the intervals over which f is concave down, the intervals over which f is concave up, and find all points of inflection of f.

   $f''(x) = -xe^{-x} + e^{-x} = (1-x)e^{-x}$. $f''(x) > 0$ when $x < 1$, and $f''(x) < 0$ when $x > 1$. Thus f is concave up when $x < 1$ and f is concave down when $x > 1$. The point $x = 1$ is a point of inflection.

Free Response Questions: Show your work!

15. For $t \geq 0$, the velocity of a particle moving along the real line is given by $v(t) = t^2 - 2t$ where t is measured in seconds.

    (a) Find the time intervals over which the velocity of the particle is positive and the time intervals over which the velocity is negative.

    $v(t) = t(t-2)$. Thus $v(t) > 0$ when $t < 0$ and $t > 2$, and $v(t) < 0$ when $0 < t < 2$.

    (b) Find the total distance traveled over the first 4 seconds.

    The total distance traveled is given by $\int_0^2 -(t^2 - 2t)dt + \int_2^4 (t^2 - 2t)dt =$ $(-\frac{1}{3}t^3 + t^2))|_0^2 + (\frac{1}{3}t^3 - t^2)|_2^4 = (\frac{-8}{3} + 4) + (\frac{64}{3} - 16) - (\frac{8}{3} - 4) = -8 + \frac{48}{3} = 8.$