Math 113 Exam 4 Fall 2015 Solutions

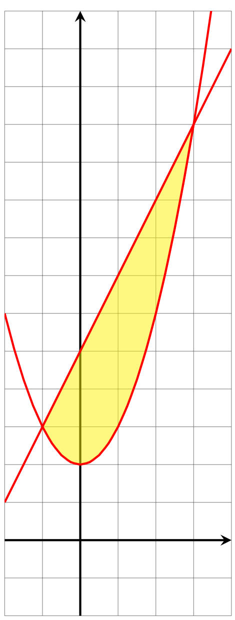
December 10, 2016

Multiple choice

BECBC CBACA

Free Response

11. The graph is:

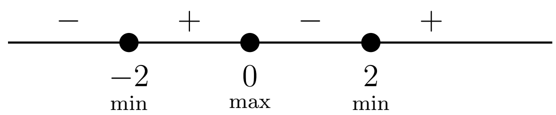


To find the points of intersection, solve $x^2 + 2 = 2x + 5$ or $x^2 - 2x - 3 = 0$ to find $x = -1$, $x = 3$. The top curve is $y = 2x + 5$ and the bottom curve is $y = x^2 + 2$. Hence the area between the two curves is given by the integral

$$\int_{-1}^{3} (2x+5) - (x^2+2)\, dx =$$

$$\int_{-1}^{3} (3 + 2x - x^2)\, dx = 44/3$$

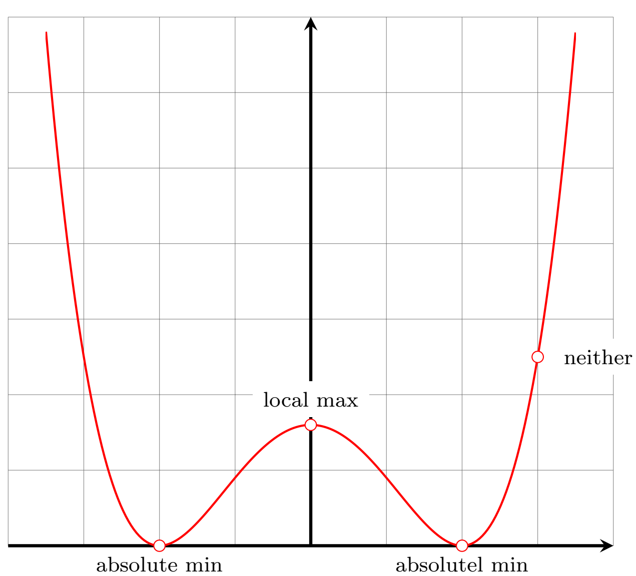
12. (a) Since $h'(x) = 4x^3 - 16x = 4x(x^2 - 4)$, the critical points are $x = \pm 2$ and $x = 0$. All of these lie in the interval $[-4, 3]$. The sign graph for the derivative is



(b) We can evaluate h at the critical points and endpoints to find

x	$f(x)$	
-4	144	**absolute max**
-2	0	**absolute min**
0	16	**local max**
2	0	**absolute min**
3	25	**neither max nor min**

Although it's not required for this problem, here is a graph of the function (the y-axis is not to scale) which shows what's going on. We don't show the point $(-4, 144)$ because it's literally off the charts!



13. (a) Factor $v(t) = 3(t^2 - 8t + 12) = 3(t-2)(t-6)$ and note that $v(t)$ is positive on $(-\infty, 2)$ and $(6, \infty)$, while $v(t)$ is negative on $(2, 6)$. Hence, the particle is moving forward in $(0, 2)$, and backward in $(2, 6)$.

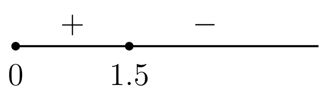
(b) The displacement is the integral of $v(t)$. This is

$$\int_0^6 (3t^2 - 24t + 36)\, dt = 0\,\mathrm{m}$$

(c) The total distance travelled is the integral of $|v(t)|$. This is

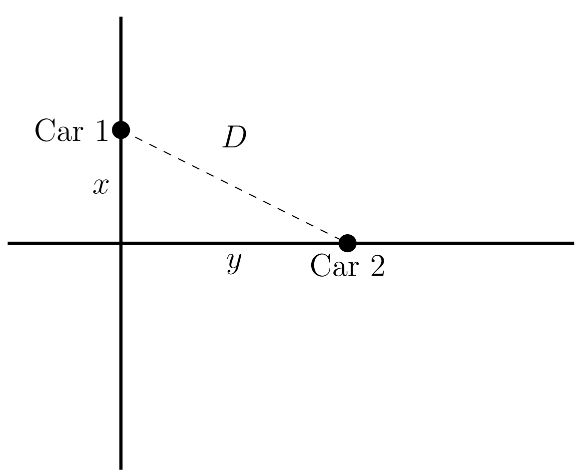
$$\int_0^2 (3t^2 - 24t + 36)\, dt + \int_2^6 -(3t^2 - 24t + 36)\, dt = 64\,\mathrm{m}$$

14. We can use the constraint $4x + y = 9$ to write $y = 9 - 4x$. Hence we need to maximize $M(x) = x^2(9-4x)$ on $(0, \infty)$. Since $M'(x) = 18x - 12x^2 = 6x(3 - 2x)$ we get critical points at $x = 0$ and $x = 3/2$. The sign graph of $M'(x)$ on $(0, \infty)$ is



By the first derivative test for absolute extrema on an interval, the function $M(x)$ has an absolute maximum on $(0, \infty)$ at $x = 3/2$, corresponding to $y = 3$. Hence, the numbers are $x = 3/2$, $y = 3$.

15. Let x be the distance of car 1 from the starting point, and let y be the distance of car 2 from the starting point. The picture looks like



The distance between the two cars is $D = \sqrt{x^2 + y^2}$. We know that

$$\frac{dx}{dt} = 20\,\text{mph}, \quad \frac{dy}{dt} = 40\,\text{mph}$$

After two hours $x = 40\,\text{miles}$, $y = 80\,\text{miles}$, so the distance D between the two cars is $40\sqrt{5}$ m. By implicit differentiation,

$$\frac{dD}{dt} = \frac{1}{2\sqrt{x^2 + y^2}}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right)$$

so at $t = 2$ we get

$$\frac{dD}{dt} = \frac{1}{80\sqrt{5}}(80 \cdot 20 + 160 \cdot 40) = \frac{100}{\sqrt{5}} = 20\sqrt{5}\,\text{mph}.$$

Of course, there is actually a much quicker way to do this problem. You can compute $x(t) = 20t$, $y(t) = 40t$, so that by the Pythagorean Theorem $D(t) = 20\sqrt{5}t$. By straightforward differentiation, $D'(t) = 20\sqrt{5}$.