Exam 4

Form A

Name: ______________________ Section and/or TA: ______________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 12 multiple choice questions and 4 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems.

Multiple Choice Questions

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10 (A) (B) (C) (D) (E)

11 (A) (B) (C) (D) (E)

12 (A) (B) (C) (D) (E)

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

Trigonometric Identities

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$
$$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$
$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

Multiple Choice Questions

1. Assuming that $\int_0^5 f(x)\,dx = 5$ and $\int_0^5 g(x) = 12$, find

$$\int_0^5 \left(3f(x) - \frac{1}{3}g(x)\right)\,dx.$$

A. 19
B. 11
C. 15
D. 17
E. 4

2. Find

$$\int_3^0 (4t^3 - 2t^2)\,dt.$$

A. 63
B. 90
C. -54
D. -63
E. -90

3. The traffic flow rate past a certain point on a highway is $f(t) = 1500 + 2500t - 270t^2$ where t is in hours and $t = 0$ is 8 AM. How many cars pass by during the time interval from 8 to 11 AM?

   A. 13320

   B. 2040

   C. 47960

   D. 34640

   E. 45920

4. Find the equation of the tangent line to $f(x) = xe^x + \cos x$ at $x = 0$.

   A. $y = x - 1$

   B. $y = 1 - x$

   C. $y = -1 - x$

   D. $y = 1 + x$

   E. None of these.

5. Let $g(x)$ be a differentiable function such that $\lim_{x\to 0} g(x) = 1$ and $\lim_{x\to 0} g'(x) = 4$. Compute the limit

$$\lim_{x\to 0} \frac{x \cos x}{g(x) - 1}.$$

A. 0
B. $\frac{1}{4}$
C. $\frac{1}{3}$
D. $\frac{1}{2}$
E. ∞

6. Find the sixth-degree Taylor polynomial approximation of $\cos x$ centered at $a = 0$. Recall that the N-th degree Taylor polynomial for $f(x)$ at $x = a$ is

$$T_N(x) = \sum_{n=0}^{N} \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

A. $x - \frac{x^3}{6} + \frac{x^5}{120}$
B. $1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}$
C. $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720}$
D. $1 + x + x^2 + x^3 + x^4 + x^5 + x^6$
E. None of the above

7. Suppose $G(x) = \int_1^{x^2} t \ln(t)\, dt$. Find $G'(2)$.

A. $2\ln(2)$

B. $4\ln(4)$

C. $1 + \ln(2)$

D. $8\ln(4) - \frac{15}{4}$

E. $16\ln(4)$

8. Find

$$\int_0^1 \frac{e^x + 1}{e^x + x}\, dx.$$

A. 1

B. -1

C. $\frac{1}{e+1} - 1$

D. $\frac{1}{(e+1)^2}$

E. $\ln(e+1)$

9. The linearization for $f(x) = \sqrt{x+3}$ at $x = 1$ is

A. $L(x) = 2 + \frac{1}{2}(x-1)$

B. $L(x) = 4 + \frac{1}{4}(x-1)$

C. $L(x) = 2 + \frac{1}{4}(x-1)$

D. $L(x) = 4 + \frac{1}{8}(x-1)$

E. $L(x) = 2 + \frac{1}{4}x - 1$

10. Find the slope of the tangent line to the curve $x^2 - xy - y^2 = 1$ at the point $(2, 1)$.

A. $\frac{3}{4}$

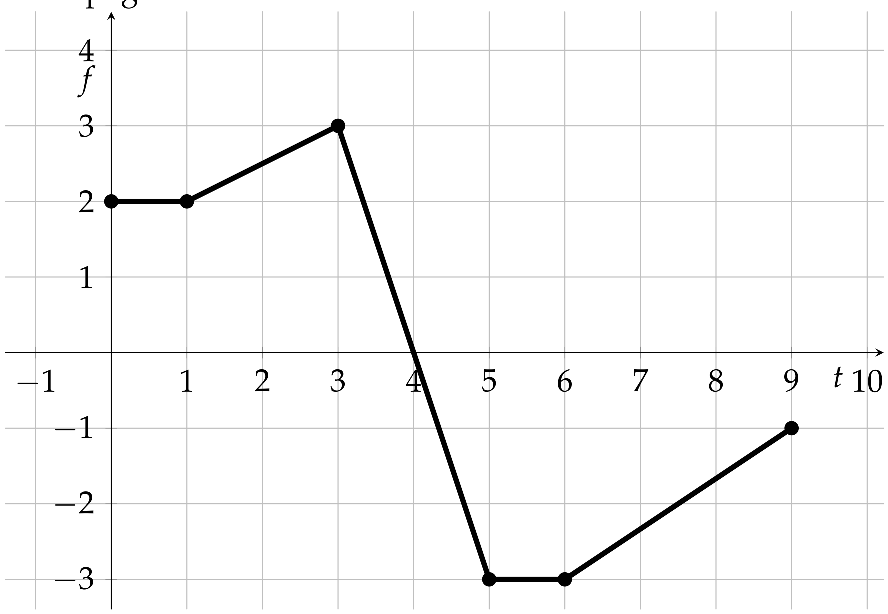
B. $\frac{3}{2}$

C. $\frac{4}{3}$

D. 0

E. $-\frac{3}{4}$

Let $F(x) = \int_0^x f(t)\,dt$ for x in the interval $[0,9]$, where $f(t)$ is the function with the graph shown in the following picture. This function F will be used for Problems 11 and 12 on this page.



11. Find $F(6)$.

A. -3

B. 3

C. 4

D. 12

E. 13

12. Which of the following statements is true?

A. F is decreasing on the interval $[3,5]$.

B. F is increasing on the interval $[6,9]$.

C. F has a local minimum at 5.

D. F has a local maximum at 4.

E. $F(5) = F(6)$.

Free Response Questions

Show all of your work

13. Compute the following antiderivatives. These are also called indefinite integrals.

(a) $\displaystyle\int \frac{2e^x}{(5e^x+3)^4}\,dx$

(b) $\displaystyle\int x\sqrt{x+1}\,dx$

(c) $\displaystyle\int \frac{\cos(x)}{(1+\sin(x))^3}\,dx$

14. (a) What does it mean for a function $f(x)$ to be continuous at a point $x = a$? Use complete sentences.

(b) Consider the piecewise defined function

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & \text{if } 0 \leq x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \leq x < 3 \\ 2x - a + b & \text{if } 3 \leq x \leq 5 \end{cases}$$

where a and b are constants. Find the values of a and b for which $f(x)$ is continuous on $[0, 5]$.

15. Let a and b be positive numbers whose product is 8. Find the minimum value of $a^2 + 2b$. Determine the values of a and b for which the minimum is attained.

16. A particle moves in a straight line so that its velocity at time t seconds is $v(t) = 12 - 2t$ feet per second.

    (a) Find the displacement of the particle over the time interval $[0, 8]$.

    (b) Find the total distance traveled by the particle over the time interval $[0, 8]$.