MA 113 — Calculus I Spring 2008
Exam 4 May 1, 2008

Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. Please, turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. Please:

1. check answers when possible,
2. clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
3. give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question. Also when appropriately record your answers at the bottom of the page.
You are to answer *two of the last three questions*. Please indicate which problem is not to be graded by crossing through its number on the table below.
Notice that there is an additional bonus problem. For its solution you can earn up to 10 points for extra credit.

Name: ______________________________

Section: __________

Last four digits of student identification number: __________

Question	Score	Total
1		9
2		8
3		9
4		8
5		8
6		8
7		10
8		9
Extra Credit		10
9		14
10		14
11		14
Free	3	3
Total		

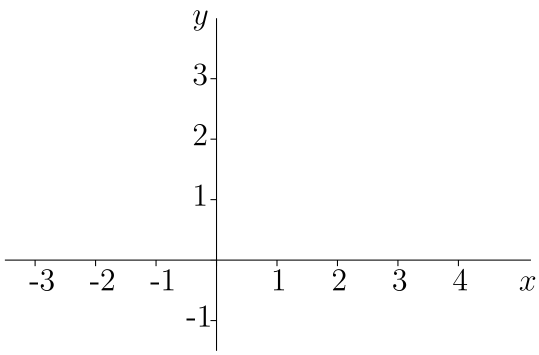
(1) Consider the function

$$f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ 2x^2 & \text{if } 0 \le x \le 1 \\ 3-x & \text{if } 1 < x \end{cases}.$$

(a) Sketch the graph of the function f.

(b) Determine all numbers a such that f is not continuous at a.

(c) Find all numbers a such that f is not differentiable at a.



(a) Discontinuities at ____________ (b) Not differentiable at ____________

(2) The cost in dollars of producing x units of a certain commodity is $C(x) = 5000 + 10x + \frac{1}{5}x^2$.

(a) Find the average rate of change of C with respect to x when the production level is changed from $x = 10$ to $x = 15$.

(b) Find the instantaneous rate of change of C with respect to x when $x = 10$.

(c) If the units, x, are increasing at the constant rate of 2 units per day, find the rate at which C is changing with respect to time (measured in days) when $x = 10$.

(a) Average change of C ____________

(b) Instantaneous rate of change of C ____________

(c) Related rate of change of C ____________

(3) Determine the following limits.

(a) $\lim_{x\to 0} \dfrac{\cos^2(x)}{4x^3-5}$.

(b) $\lim_{x\to \infty} \dfrac{7x^4+x^2+\sqrt{x}}{1+x^4}$.

(c) $\lim_{x\to 2} \dfrac{x^2+x-6}{x^2-4}$.

(a) **Answer to Q.3(a)** ____________________

(b) **Answer to Q.3(b)** ____________________

(c) **Answer to Q.3(c)** ____________________

(4) Determine the following limits by interpreting them as a derivative or integral.

(a) $\displaystyle\lim_{h\to 0} \frac{\sqrt[4]{16+h} - \sqrt[4]{16}}{h}.$

(b) $\displaystyle\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \sqrt{3 + \frac{i}{n}}.$

(a) **Answer to Q.4(a)** ____________________________

(b) **Answer to Q.4(b)** ____________________________

(5) Find the equation of the tangent line to the curve defined by $x^3 + 2xy + y^3 = 13$ at the point $P(1, 2)$. Give your final answer in the form $y = mx + b$.

Equation is: $y =$ ________________

(6) Find the absolute minimum value and the absolute maximum value of the function $f(x) = x + \dfrac{4}{x}$ on the interval $[1, 5]$.

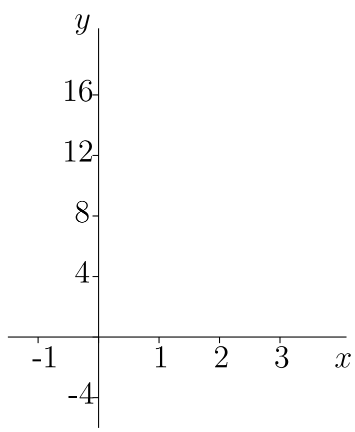
Absolute maximum value: ____________________

Absolute minimum value: ____________________________

(7) (a) Determine the points where the two curves $y = 8x$ and $y = x^4$ meet.

(b) Sketch the region bounded by the two curves.

(c) Find the area of this region.



Points of intersection: ______________

Area of the region ______________

(8) Evaluate the following integrals.

(a) $\displaystyle\int (2+x^3)^2\,dx.$

(b) $\displaystyle\int x \cdot \sqrt[3]{7-6x^2}\,dx.$

(c) $\displaystyle\int_0^{\pi/3} \frac{\sin(t)}{\cos^2(t)}\,dt.$

(a) **Answer to Q.8(a)** ____________________

(b) **Answer to Q.8(b)** ____________________

(c) **Answer to Q.8(c)** ____________________

Extra Credit Problem.

Determine if the following statements are true or false. No justification is required.
Each correct answer is worth 2 points. Each false answer leads to a deduction of 1 point.
However, your total score for this problem will not be below zero.

(a) Suppose the function f is continuous on $[1, 5]$ and differentiable on $(1, 5)$.
If $f(1) = 5$ and $f(5) = 20$, then $f'(t) \geq 4$ for at least one t in $(1, 5)$.

Answer: True ☐ **False** ☐

(b) Suppose the function f is differentiable on the set of real numbers.
Then f is continuous.

Answer: True ☐ **False** ☐

(c) The function $f(x) = \displaystyle\int_1^x |t|\, dt$ is differentiable.

Answer: True ☐ **False** ☐

(d) If $f'(a) = 0$, then f has a local maximum or a local minimum at a.

Answer: True ☐ **False** ☐

(e) If f is a continuous function on the open interval $(1, 10)$, then f has an absolute maximum at some point c in $(1, 10)$.

Answer: True ☐ **False** ☐

Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front of the exam.

(9) (a) State both parts of the Fundamental Theorem of Calculus. Use complete sentences.

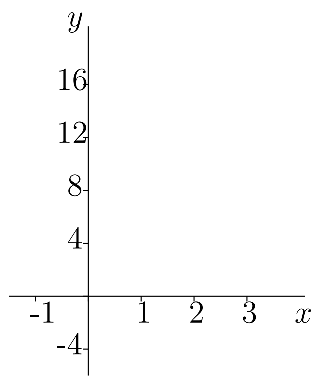
(b) Consider the function f on $[1,\infty)$ defined by $f(x) = \int_1^x \sqrt{t^5 - 1}\, dt$. Argue that f is increasing.

(c) Find the derivative of the function $g(x) = \int_1^{x^3} \sqrt{t^5 - 1}\, dt$ on $(1,\infty)$.

(10) Consider the region bounded by the curves $y = 8x$ and $y = 4x^2$.

(a) Sketch the region.

(b) Find the volume of the solid obtained by revolving the region about the y-axis.



(b) Volume: ______________

(11) A company wants to produce cylindrical beer glasses that can hold 0.5 liter beer. It costs 1 cent per square centimeter to manufacture the side of the glass and $\frac{4}{\pi}$ cents per square centimeter to manufacture its bottom. Find the dimensions (in centimeters) and the cost of the cheapest such glass.
(Note that 1 liter equals 1000 cm^3.)

Height: ____________ cm   Radius of the base: ____________ cm

Cost: ____________ cents