
Assuming that $\int_0^5 f(x)\,dx = 5$ and $\int_0^5 g(x) = 12$, find

$$\int_0^5 \left(3f(x) - \frac{1}{3}g(x)\right)\,dx.$$

- A. 11
- B. 4
- C. 15
- D. 19
- E. 17
- F. None of the above

Correct Answers:

- `A`

If f and g are continuous functions with $f(9) = 6$ and $\lim_{x\to 9}[2f(x) - g(x)] = 9$, find $g(9)$.

- A. 24
- B. 3
- C. 21
- D. 15
- E. 12
- F. None of the above

Correct Answers:

- `B`

Find the equation to the tangent line to $y = \frac{\sqrt{x}}{x+6}$ at $\left(4, \frac{2}{10}\right)$.

- A. $y = 0.2 + (x-4)$
- B. $y = \frac{1}{5} + \frac{1}{200}(x-4)$
- C. $y = \frac{1}{200} + \frac{2}{10}(x-4)$
- D. $y = 0.2 + \frac{1}{20}(x-4)$
- E. $y = 0.2 + \frac{6-x}{2\sqrt{x}(x+6)^2}(x-4)$
- F. None of the above

Correct Answers:

- `B`

The function $f(x)$ is given below.

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } 0 \le x < 2 \\ ax^2 - bx + 2 & \text{if } 2 \le x < 3 \\ 2x - 3a - b & \text{if } 3 \le x \le 5 \end{cases}$$

where a and b are constants. Find the values of a and b for which $f(x)$ is continuous on $[0,5]$.

- A. $a = 0$, $b = -2$
- B. $a = 0$, $b = -1$
- C. $a = \frac{1}{2}$, $b = 0$
- D. $a = \frac{1}{4}$, $b = -\frac{1}{2}$
- E. $a = -\frac{1}{4}$, $b = \frac{1}{2}$
- F. None of the above

Correct Answers:

- `D`

If the tangent line to $y = f(x)$ at $(8,4)$ passes through the point $(4,-32)$, find $f'(8)$.

- A. $f'(8) = -9$
- B. $f'(8) = 19$
- C. $f'(8) = 9$
- D. $f'(8) = 29$
- E. $f'(8) = 34$
- F. None of the above

Correct Answers:

- `C`

If $h(x) = g\left[(f(x))^2\right]$, $g'(-6) = 5$, $g'(4) = -2$, $f(2) = 2$ and $f'(2) = -3$, find $h'(2)$.

- A. 12
- B. -8
- C. -12
- D. 5
- E. 24
- F. None of the above

Correct Answers:

- `E`

If $f(2) = 7$ and $f'(2) = -5$, find $\frac{d}{dx} \frac{f(x)}{x^2+1}\Big|_{x=2}$.

- A. $\frac{1}{25}$
- B. $-\frac{53}{25}$
- C. $\frac{3}{25}$
- D. -29
- E. $-\frac{33}{5}$
- F. None of the above

Correct Answers:

- `B`

Find f' in terms of g' if

$$f(x) = x^5 g(x).$$

- A. $f'(x) = 5x^4 g(x) + x^5 g'(x)$
- B. $f'(x) = x^5 g(x) + 5x^4 g'(x)$
- C. $f'(x) = 5x^4 g'(x)$
- D. $f'(x) = x^5 g'(x) - 5x^4 g(x)$
- E. $f'(x) = 5x^4 + g'(x)$
- F. None of the above

Correct Answers:

- `A`

The linearization for $f(x) = \frac{x}{1+x^2}$ at $x = 2$ is

- A. $L(x) = \frac{3}{25}x - \frac{1}{25}$
- B. $L(x) = \frac{2}{5}x$
- C. $L(x) = \frac{2}{5}$
- D. $L(x) = -\frac{3}{25}x + \frac{16}{25}$
- E. $L(x) = x + 2$
- F. None of the above

Correct Answers:

- `D`

Find the equation of the tangent line to the curve $x^2 - xy - y^2 = 1$ at the point $(2,1)$.

- A. $y = \frac{3}{4}x + \frac{5}{2}$
- B. $y = -\frac{3}{4}x + \frac{5}{2}$
- C. $y = \frac{3}{4}x - \frac{1}{2}$
- D. $y = 1$
- E. $y = -\frac{3}{4}x + \frac{1}{2}$
- F. None of the above

Correct Answers:

- `C`

Find the absolute maximum value of the function $f(x) = x\sqrt{4x - x^2}$ on the interval $[0,4]$.

- A. $3\sqrt{7}$
- B. $\frac{52}{10}$
- C. $4\sqrt{2}$
- D. $3\sqrt{3}$
- E. 3
- F. None of the above

Correct Answers:

- `D`

The function $f(x) = 7x^2 - x + 5$ satisfies the hypotheses of the Mean Value Theorem on the interval $[-1,7]$. Find all values of c that satisfy the conclusion of the theorem.

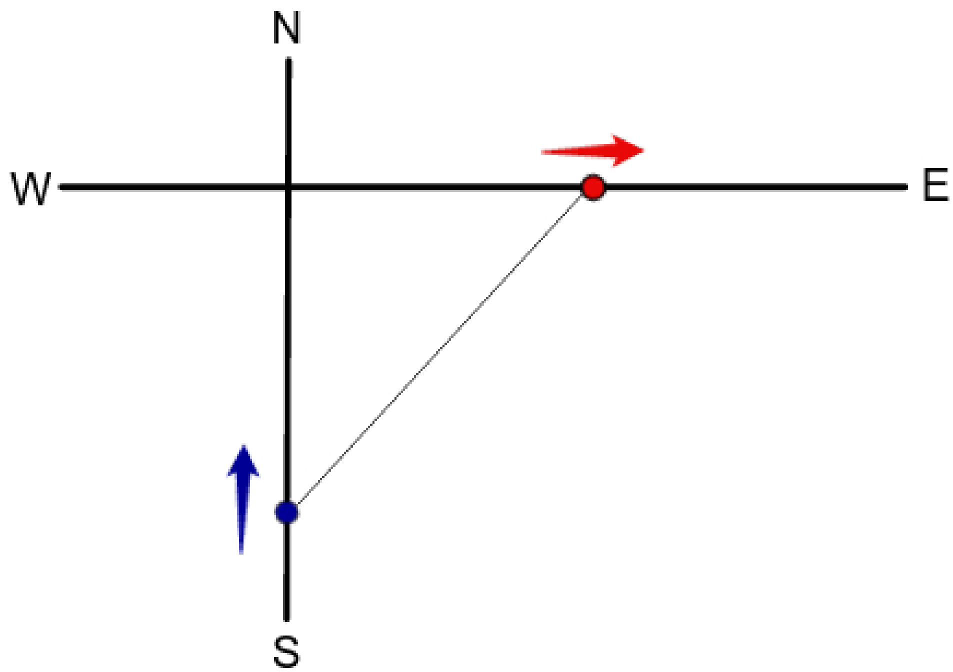
- A. 2, 4
- B. 4
- C. 3
- D. 3, 4
- E. 2, 3
- F. None of the above

Correct Answers:

- `C`

---

13. **(5 points)** `Library/Valdosta/APEX_Calculus/4.2/APEX_4.2_6.pg`



Suppose a police officer is 1/2 mile south of an intersection, driving north towards the intersection at 40 mph. At the same time, another car is 1/2 mile east of the intersection, driving east (away from the intersection) at an unknown speed. The officer's radar gun indicates 20 mph when pointed at the other car (that is, the straight-line distance between the officer and the other car is increasing at a rate of 20 mph). What is the speed of the other car?

Speed = _______ mph.

Now suppose that the officer's radar gun indicates -20 mph instead (that is, the straight-line distance is decreasing at a rate of 20 mph). What is the speed of the other car this time?

Speed = _______ mph.

Solution: *(Instructor solution preview: show the student solution after due date.)*

Solution:

The two cars and the intersection form a right triangle. Let $a =$ distance from the officer's car to the intersection (along the north-south street), and $b =$ distance from the intersection to the other car (along the west-east street). Let $c =$ straight-line distance from the officer's car to the other car. Then by the Pythagorean Theorem, $a^2 + b^2 = c^2$. Take time-derivatives of both sides of this equation.

$$\begin{aligned} \frac{d}{dt}\left[a^2+b^2\right] &= \frac{d}{dt}\left[c^2\right] \\ 2a\frac{da}{dt}+2b\frac{db}{dt} &= 2c\frac{dc}{dt} \end{aligned}$$

Then plug in the known information and solve. We have $\frac{da}{dt} = -40$, which is negative because the distance from the officer to the intersection is decreasing. We also have $\frac{dc}{dt} = 20$. We can find the distance between the cars at this specific moment: $c = \sqrt{(1/2)^2 + (1/2)^2} = \frac{\sqrt{2}}{2}$.

$$2\left(\tfrac{1}{2}\right)(-40) + 2\left(\tfrac{1}{2}\right)\frac{db}{dt} = 2\left(\tfrac{\sqrt{2}}{2}\right)(20)$$

$$\frac{db}{dt} = 20\sqrt{2} + 40 \approx 68.2842712474619.$$

For the second part, $\frac{dc}{dt} = -20$.

$$2\left(\tfrac{1}{2}\right)(-40) + 2\left(\tfrac{1}{2}\right)\frac{db}{dt} = 2\left(\tfrac{\sqrt{2}}{2}\right)(-20)$$

$$\frac{db}{dt} = -20\sqrt{2} + 40 \approx 11.7157287525381.$$

Correct Answers:

- `68.2842712474619`
- `11.7157287525381`

---

14. **(5 points)** `Library/Valdosta/APEX_Calculus/6.7/APEX_6.7_9.pg`

Evaluate the limit, using L'Hôpital's Rule.
Enter **INF** for ∞, **-INF** for $-\infty$, or **DNE** if the limit does not exist, but is neither ∞ nor $-\infty$.

$\lim_{t\to -4} \frac{t^2-16}{3t^2+7t-20} =$ ____

Solution: *(Instructor solution preview: show the student solution after due date.)*

$$\lim_{t\to -4} \frac{t^2-16}{3t^2+7t-20} = \lim_{t\to -4} \frac{2t}{6t+7} = \frac{-8}{-17}$$

Correct Answers:

- `0.470588`

---

15. **(5 points)** `Library/UCSB/Stewart5_4_4/Stewart5_4_4_31.pg`

Find the limit. Use l'Hospital's Rule if appropriate. Use INF to represent positive infinity, NINF for negative infinity, and D for the limit does not exist.

$\lim_{x\to\infty} \frac{7x}{3\ln(1+2e^x)} =$ ________

Correct Answers:

- `2.33333333333333`

---

16. **(5 points)** `Library/Valdosta/APEX_Calculus/4.3/APEX_4.3_7.pg`

Find the maximal area of a right triangle with hypotenuse of length 8.

____

Solution: *(Instructor solution preview: show the student solution after due date.)*

Solution:

Let x, y be the lengths of the legs of the triangle. By the Pythagorean Theorem, $x^2 + y^2 = 8^2$. Solving for y, we get:

$$y = \sqrt{64 - x^2}$$

The fundamental equation (to be maximized) is the area of the triangle, $A = \frac{1}{2}xy$. Substitute $y = \sqrt{64-x^2}$ and find the critical number(s).

$$A = \tfrac{1}{2}x\sqrt{64-x^2}$$

$$A' = \tfrac{1}{2}\left(\frac{-x^2}{\sqrt{64-x^2}} + \sqrt{64-x^2}\right)$$

$$\tfrac{1}{2}\left(\frac{-x^2}{\sqrt{64-x^2}} + \sqrt{64-x^2}\right) =$$

$$\begin{aligned} -x^2 + (64-x^2) &= 0 \\ 2x^2 &= 64 \\ x &= \pm\sqrt{32} \end{aligned}$$

Since lengths are required to be positive, we choose $x = \sqrt{32}$. The corresponding y value is:

$$y = \sqrt{64-x^2} = \sqrt{64-32} = \sqrt{32}.$$

Therefore, the maximal area is:

$$A = \frac{1}{2}xy = \frac{1}{2}\sqrt{32}\cdot\sqrt{32} = 16$$

Correct Answers:

- `16`

---

17. (5 points) `Library/UMN/calculusStewartCCC/s_4_8_31.pg`

Find f if $f''(x) = \sin x + \cos x$, $f'(0) = 8$, and $f(0) = 2$.

Answer: $f(x) =$ ____________________

Correct Answers:

- `-[sin(x)]-cos(x)+9*x+3`

---

18. (5 points) `Library/UMN/calculusStewartCCC/s_5_4_25.pg`

If $f(1) = 11$, f' is continuous, and $\int_1^8 f'(t)\,dt = 23$, what is the value of $f(8)$?

Answer: __________

Correct Answers:

- `11+23`

---

19. (5 points) `Library/Wiley/setAnton_Section_5.6/Anton_5_6_Q63.pg`

Let $F(x) = \int_{10}^{x} \sqrt{t^2+21}\,dt$. Find

(a) $F(10) =$ __________

(b) $F'(10) =$ __________

(c) $F''(10) =$ __________

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

(a) $F(10) = \int_{10}^{10} \sqrt{t^2+21}\,dt = 0$.

(b) $F'(x) = \dfrac{d}{dx}\displaystyle\int_{10}^{x} \sqrt{t^2+21}\,dt = \sqrt{x^2+21}$, $F'(10) = 11$.

(b) $F''(x) = \frac{d}{dx}\left[\sqrt{x^2+21}\right] = \frac{x}{\sqrt{x^2+21}}$, $F''(10) = \frac{10}{11}$.

Correct Answers:

- `0`

- 11
- 10/11

---

20. **(5 points)** Library/UMN/calculusStewartCCC/s_5_5_67.pg

If f is continuous and $\int_0^{10} f(x)\,dx = 50$, find $\int_0^5 f(2x)\,dx$.

Answer: ____________

Correct Answers:

- 1/2*50