Exam 4

Solutions

Multiple Choice Questions

1. Find the values of A and B such that the function

$$f(x) = \begin{cases} -2x^2+5, & x \leq -1 \\ Ax+B, & -1 < x < 2 \\ 2x^2-3, & 2 \leq x \end{cases}$$

is continuous.

A. $A = \frac{2}{3}$, $B = \frac{11}{3}$

B. $A = 2$, $B = 5$

C. $A = 2$, $B = 1$

D. $A = 1$, $B = 4$

E. $A = \frac{1}{2}$, $B = 4$

2. Suppose that f is a differentiable function on $(0, 4)$, that $f'(x) > 0$ for x in each of the intervals $(0, 1)$, $(1, 2)$ and $(3, 4)$ and that $f'(x) < 0$ on the interval $(2, 3)$.

Select the correct statement.

A. f has a local minimum at 1 and no local maximum.

B. f has a local minimum at 2 and a local maximum at 3.

C. f has a local maximum at 2 and a local minimum at 3.

D. f has local minima at 1 and 3 and a local maximum at 2.

E. f has local maxima at 1 and 3 and a local minimum at 2.

3. $\int_0^{\pi/4} \sin^3 x \cos x \, dx =$ ________

   A. $\frac{1}{4}$

   B. $\frac{3}{50}$

   C. $\frac{1}{6}$

   D. $\frac{66}{1000}$

   E. $\frac{1}{16}$

4. Select the correct statement from below about the function $f(x) = \dfrac{x^2 + 2x - 8}{x - 2}$.

   A. $f(2) = 6$

   B. The function has a jump discontinuity at $x = 2$.

   C. The function is continuous at $x = 2$.

   D. The function has a removable discontinuity at $x = 2$.

   E. The function has an infinite discontinuity (vertical asymptote) at $x = 2$.

5. If $f(x) = \int_0^{5x^2} e^{-t^2}\,dt$, find $f'(x)$.

   A. $f'(x) = e^{-x^2}$

   B. $f'(x) = 10xe^{-25x^4}$**.**

   C. $f'(x) = 5x^2e^{-x^2}$

   D. $f'(x) = e^{-25x^4}$

   E. $f'(x) = 5x^2e^{-25x^4}$

6. Find the limit

$$\lim_{x\to 0} \frac{e^x - x - 1}{2 - 2\cos x}.$$

   A. 0

   B. $-\frac{1}{2}$

   C. $\frac{1}{2}$

   D. 1

   E. Does not exist

7. If $2x + y = 9$, what is the smallest possible value of $4x^2 + 3y^2$?

   A. 60.00

   B. 60.25

   C. 60.50

   D. 60.75

   E. 61.00

8. If $f(1) = 6$, f' is continuous, and $\int_1^8 f'(t)dt = 14$, what is the value of $f(8)$?

   A. 8

   B. 10

   C. 18

   D. 20

   E. 22

9. The linearization for $f(x) = \sqrt{x+3}$ at $x = 1$ is

   A. $L(x) = 2 + \frac{1}{4}x - 1$

   B. $L(x) = 2 + \frac{1}{4}(x-1)$

   C. $L(x) = 2 + \frac{1}{2}(x-1)$

   D. $L(x) = 4 + \frac{1}{4}(x-1)$

   E. $L(x) = 4 + \frac{1}{8}(x-1)$

10. Find the slope of the tangent line to the curve $x^2 - xy - y^2 = 1$ at the point $(2, -3)$.

   A. $-\frac{7}{4}$

   B. $-\frac{1}{4}$

   C. $\frac{1}{8}$

   D. $\frac{1}{4}$

   E. $\frac{7}{8}$

11. Find the equation of the tangent line to $g(x) = \dfrac{2x}{1+x^2}$ at $x = 3$.

A. $y = -\dfrac{4}{25}x + \dfrac{3}{25}$

B. $y = \dfrac{1}{3}x - \dfrac{2}{5}$

C. $y = 2x - \dfrac{27}{5}$

D. $y = 16x - \dfrac{474}{10}$

E. $y = -\dfrac{4}{25}x + \dfrac{27}{25}$.

12. Find the derivative of $f(x) = x^2 e^{\cos(2x)}$.

A. $f'(x) = -2(x\sin(2x) - 1)e^{\cos(2x)}$

B. $f'(x) = 2\left(x - x^2\sin(2x)\right)e^{\cos(2x)}$

C. $f'(x) = (x\sin(2x) - 1)e^{\cos(2x)}$

D. $f'(x) = 2xe^{-2\sin(2x)}$

E. $f'(x) = -2x\sin(2x)e^{\cos(2x)}$

13. Given that $f''(x) = 6x - 4$, $f'(1) = 2$, and $f(2) = 10$, find $f(x)$.

A. $f(x) = x^3 - 2x^2 + 10$

B. $f(x) = x^3 - 2x^2 + x + 8$

C. $f(x) = x^3 - 2x^2 + 2x + 6$

D. $f(x) = x^3 - 2x^2 + 3x + 4$

E. $f(x) = x^3 - 2x^2 + 4x + 2$

14. Find $\int_x^{x^2} \sin(2t)\,dt$.

A. $\frac{1}{2}\cos(2x^2) - \frac{1}{2}\cos(2x)$

B. $-\cos(2x^2) + \cos(2x)$

C. $\cos(2x^2 - 2x)$

D. $\cos(2x^2) - \cos(2x)$

E. $-\frac{1}{2}\cos(2x^2) + \frac{1}{2}\cos(2x)$

Free Response Questions
Show all of your work

15. Compute the following general antiderivatives. These are also called indefinite integrals.

(a) $\displaystyle\int \frac{x}{1+x^2}\,dx$

Solution: Use the Method of Substitution. Let $u = 1 + x^2$, then $du = 2x\,dx$. Thus,

$$\int \frac{x}{1+x^2}\,dx = \frac{1}{2}\int \frac{1}{1+x^2}\,2x\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln u + C = \frac{1}{2}\ln(1+x^2) + C.$$

(b) $\displaystyle\int \frac{(\arctan x)^3}{1+x^2}\,dx$

Solution: Use the Method of Substitution. Let $u = \arctan x$, then $du = \dfrac{1}{1+x^2}\,dx$. Then,

$$\int \frac{(\arctan x)^3}{1+x^2}\,dx = \int u^3\,du = \frac{1}{4}u^4 + C = \frac{1}{4}(\arctan x)^4 + C.$$

(c) $\displaystyle\int x\sqrt{x-1}\,dx$

Solution: Use the Method of Substitution. Let $u = x - 1$, then $du = dx$ and $x = u + 1$. Thus,

$$\begin{aligned}
\int x\sqrt{x-1}\,dx &= \int (u+1)\sqrt{u}\,du \\
&= \int u^{\frac{3}{2}} + u^{\frac{1}{2}}\,du \\
&= \frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}} + C \\
&= \frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + C
\end{aligned}$$

16. The tangent line to the graph of a function $f(x)$ at the point $x = 1$ is $y = 5x + 2$.

(a) What is $f(1)$?

Solution: $f(1)$ has to be the same as the value of y on the tangent line at $x = 1$, so $f(1) = 5(1) + 2 = 7$.

(b) What is $f'(1)$?

Solution: $f'(1)$ is the slope of the tangent line to $f(x)$ at $x = 1$, so $f'(1) = 5$.

(c) If $g(x) = f(x^5)$, then find $g'(1)$. Show your work.

Solution: By the Chain Rule $g'(x) = f'(x^5)(5x^4)$, so

$$g'(1) = f'(1^5)(5 \cdot 1^4) = 5 \times 5 = 25.$$

17. The velocity of a particle moving on a straight line is

$$v(t) = 3t^2 - 24t + 36 \text{ meters/second,}$$

for $0 \le t \le 6$.

(a) Find the displacement of the particle over the time interval $0 \le t \le 6$. Show your work.

Solution: The displacement is simply the integral of $v(t)$ over the interval:

$$\begin{aligned}\text{displacement} &= \int_0^6 (3t^2 - 24t + 36)\,dt \\ &= t^3 - 12t^2 + 36t\Big|_0^6 \\ &= 216 - 432 + 216 \\ &= 0 \text{ meters}\end{aligned}$$

(b) Find the total distance traveled by the particle over the time interval $0 \le t \le 6$. Show your work.

Solution: To find the total distance traveled by the particle, we need to find if the velocity changes sign in the interval $[0, 6]$. Setting $3t^2 - 24t + 36 = 0$ we find $t = 2$ and $t = 6$. Thus, the velocity changes sign at $t = 2$. Therefore,

$$\begin{aligned}\text{distance} &= \left|\int_0^2 (3t^2 - 24t + 36)\,dt\right| + \left|\int_2^6 (3t^2 - 24t + 36)\,dt\right| \\ &= \left|t^3 - 12t^2 + 36t\Big|_0^2\right| + \left|t^3 - 12t^2 + 36t\Big|_2^6\right| \\ &= |8 - 24 + 72 - 0| + |216 - 432 + 216 - (8 - 24 + 72)| \\ &= 56 + |-56| \\ &= 112 \text{ meters}\end{aligned}$$