Worksheet # 3: Limits: A Numerical and Graphical Approach

1. Comprehension check:

   (a) In words, describe what "$\lim_{x\to a} f(x) = L$" means.

   (b) In words, what does "$\lim_{x\to a} f(x) = \infty$" mean?

   (c) Suppose $\lim_{x\to 1} f(x) = 2$. Does $f(1) = 2$?

   (d) Suppose $f(1) = 2$. Does $\lim_{x\to 1} f(x) = 2$?

2. Compute the value of the following functions near the given $x-$value. Use this information to guess the value of the limit of the function (if it exist) as x approaches the given value.

   (a) $f(x) = (x-2)^3 - 1$, $x = 1$

   (b) $f(x) = \frac{4x^2-9}{2x-3}$, $x = \frac{3}{2}$

   (c) $f(x) = \frac{x}{|x|}$, $x = 0$

   (d) $f(x) = 2^{x-1} + 1$, $x = 1$

   (e) $f(x) = \frac{x^2-3x+2}{x^2+x-6}$, $x = 2$

3. Let $f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ x-1 & \text{if } 0 < x \text{ and } x \neq 2 \\ -3 & \text{if } x = 2 \end{cases}$.

   (a) Sketch the graph of f.

   (b) Compute the following:

      i. $\lim_{x\to 0^-} f(x)$

      ii. $\lim_{x\to 0^+} f(x)$

      iii. $\lim_{x\to 0} f(x)$

      iv. $f(0)$

      v. $\lim_{x\to 2^-} f(x)$

      vi. $\lim_{x\to 2^+} f(x)$

      vii. $\lim_{x\to 2} f(x)$

      viii. $f(2)$

4. In the following, sketch the functions and use the sketch to compute the limit.

   (a) $\lim_{x\to 3} \pi$

   (b) $\lim_{x\to \pi} x$

   (c) $\lim_{x\to a} |x|$

   (d) $\lim_{x\to 3} 2^x$

5. Compute the following limits or explain why they fail to exist:

   (a) $\displaystyle\lim_{x\to -3^+} \frac{x+2}{x+3}$

   (b) $\displaystyle\lim_{x\to -3^-} \frac{x+2}{x+3}$

   (c) $\displaystyle\lim_{x\to -3} \frac{x+2}{x+3}$

   (d) $\displaystyle\lim_{x\to 0^-} \frac{1}{x^3}$

6. In the theory of relativity, the mass of a particle with velocity v is:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where m_0 is the mass of the particle at rest and c is the speed of light. What happens as $v \to c^-$?

7. Let $f(x) = \begin{cases} 2x + 2 & \text{if } x > -2 \\ a & \text{if } x = -2 \\ kx & \text{if } x < -2 \end{cases}$. Find k and a so that $\lim_{x \to -2} f(x) = f(-2)$.