Worksheet # 7: Trigonometric Functions and Limits

1. Let $\pi \leq \alpha \leq \frac{3\pi}{2}$ and $|\cos(\alpha)| = \frac{5}{12}$. Find $\sin(\alpha)$
2. Evaluate (without using your calculator) $\tan(\sin^{-1}(-3/5))$.
3. Let $f(x) = \sin(x)$ for x satisfying $\pi/2 \leq x \leq 3\pi/2$. Sketch the graphs of f and f^{-1}.
4. Evaluate $\cos^{-1}(\cos(5\pi/2))$.
5. Only one of the following statements is true. Which one?
   (a) $\cos(\cos^{-1}(x)) = x$
   (b) $\cos^{-1}(\cos(x)) = x$.
6. Let $f(x) = 1 + x^2 \sin(1/x)$ for $x \neq 0$. Find two simpler functions g and h so that we can use the squeeze theorem to show $\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x) = \lim_{x\to 0} h(x)$. Give the common value of the limits.
7. Let n be a positive integer, evaluate the limit $\lim_{t\to 0} \dfrac{\tan(nt)}{t \sec t}$
8. Evaluate the limit $\lim_{h\to 0} \dfrac{1-\cos h}{h^2}$. **Hint:** Multiply and divide by $1 + \cos h$
9. Evaluate the limit $\lim_{t\to 0} \dfrac{\sqrt{1-\cos t}}{t}$
10. Evaluate the limit $\lim_{t\to \frac{\pi}{2}} \dfrac{1-\cos t}{t}$
11. Evaluate the limit $\lim_{t\to 0} \dfrac{\sin(2t)(1-\cos(3t))}{t^2}$
12. Let k and m be positive constants, find $\lim_{x\to 0} \dfrac{\sec(kx) - 1}{mx}$

    The following identity may be useful for the next problems.

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y) \tag{1}$$

13. Use equation (1), to simplify the limit

$$\lim_{h\to 0} \frac{\cos(x+h) - \cos(x)}{h}$$

14. Evaluate the limits:
    (a) Evaluate the limit $\lim_{t\to 0} \dfrac{t^2}{\sin t}$.
    (b) Find the limit $\lim_{t\to 0} \dfrac{\cos(5t) - \cos^2(5t)}{t}$.
    (c) Evaluate the limit $\lim_{x\to 0} \dfrac{\tan(11x)}{5x}$.
    (d) Evaluate the limit $\lim_{x\to 0} \dfrac{\cos(2x) - 1}{\cos(x) - 1}$ **Hint:** Use equation (1)
    (e) Evaluate the limit $\lim_{x\to 0} \dfrac{1-\cos(3x)}{x^2}$. **Hint:** Multiply and divide by $1 + \cos(3x)$
    (f) Evaluate the limit $\lim_{x\to 0} \dfrac{\cos x - \cos 3x}{x^2}$. **Hint:** Use equation (1) to rewrite $\cos(3x)$ as $\cos(x + 2x)$