Worksheet # 11: Product and Quotient Rules

1. Show by way of example that, in general,

$$\frac{d}{dx}(f \cdot g) \neq \frac{df}{dx} \cdot \frac{dg}{dx}$$

and

$$\frac{d}{dx}\left(\frac{f}{g}\right) \neq \frac{\frac{df}{dx}}{\frac{dg}{dx}}.$$

2. State the quotient and product rule and be sure to include all necessary hypotheses.

3. Compute the first derivative of each of the following:

   (a) $f(x) = \dfrac{\sqrt{x}}{x-1}$

   (b) $f(x) = (3x^2 + x)e^x$

   (c) $f(x) = \dfrac{e^x}{2x^3}$

   (d) $f(x) = (x^3 + 2x + e^x)\left(\dfrac{x-1}{\sqrt{x}}\right)$

   (e) $f(x) = \dfrac{2x}{4+x^2}$

   (f) $f(x) = \dfrac{ax+b}{cx+d}$

   (g) $f(x) = \dfrac{(x^2+1)(x^3+2)}{x^5}$

   (h) $f(x) = (x-3)(2x+1)(x+5)$

4. Let $f(x) = (3x-1)e^x$. For which x is the slope of the tangent line to f positive? Negative? Zero?

5. Calculate the derivatives of the following functions in the two ways that are described.

   (a) $f(x) = x^5$

      i. using the power rule

      ii. using the product rule by considering the function as $f(x) = x^2 \cdot x^3$

   (b) $g(x) = (x^2+1)(x^4-1)$

      i. by distributing the two factors and using the power rule

      ii. by using the product rule

6. Find an equation of the tangent line to the given curve at the specified point. Sketch the curve and the tangent line to check your answer.

   (a) $y = x^2 + \dfrac{e^x}{x^2+1}$ at the point $x = 3$.

   (b) $y = 2xe^x$ at the point $x = 0$.

7. Suppose that $f(2) = 3$, $g(2) = 2$, $f'(2) = -2$, and $g'(2) = 4$. For the following functions, find $h'(2)$.

   (a) $h(x) = 5f(x) + 2g(x)$

   (b) $h(x) = f(x)g(x)$

   (c) $h(x) = \dfrac{f(x)}{g(x)}$

   (d) $h(x) = \dfrac{g(x)}{1+f(x)}$