Worksheet # 13: Chain Rule

1. (a) Carefully state the chain rule using complete sentences.

   (b) Suppose f and g are differentiable functions so that $f(2) = 3$, $f'(2) = -1$, $g(2) = \dfrac{1}{4}$, and $g'(2) = 2$. Find each of the following:

      i. $h'(2)$ where $h(x) = \sqrt{[f(x)]^2 + 7}$.

      ii. $l'(2)$ where $l(x) = f(x^3 \cdot g(x))$.

2. Given the following functions: $f(x) = \sec(x)$, and $g(x) = x^3 - 2x + 1$. Find:

   (a) $f(g(x)) =$

   (b) $f'(x) =$

   (c) $g'(x) =$

   (d) $f'(g(x)) =$

   (e) $(f \circ g)'(x) =$

3. Differentiate each of the following and simplify your answer.

   (a) $f(x) = \sqrt[3]{2x^3 + 7x + 3}$

   (b) $g(t) = \tan(\sin(t))$

   (c) $h(u) = \sec^2(u) + \tan^2(u)$

   (d) $f(x) = xe^{(3x^2+x)}$

   (e) $g(x) = \sin(\sin(\sin(x)))$

4. Find an equation of the tangent line to the curve at the given point.

   (a) $f(x) = x^2e^{3x}$, $x = 2$

   (b) $f(x) = \sin(x) + \sin^2(x)$, $x = 0$

5. Compute the derivative of $\frac{x}{x^2+1}$ in two ways:

   (a) Using the quotient rule.

   (b) Rewrite the function $\frac{x}{x^2+1} = x(x^2+1)^{-1}$ and use the product and chain rule.

   Check that both answers give the same result.

6. If $h(x) = \sqrt{4 + 3f(x)}$ where $f(1) = 7$ and $f'(1) = 4$, find $h'(1)$.

7. Let $h(x) = f \circ g(x)$ and $k(x) = g \circ f(x)$ where some values of f and g are given by the table

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	4	4	-1	-1
2	3	4	3	-1
3	-1	-1	3	-1
4	3	2	2	-1

   Find: $h'(-1), h'(3)$ and $k'(2)$.

8. Find all x values so that $f(x) = 2\sin(x) + \sin^2(x)$ has a horizontal tangent at x.

9. Comprehension check for derivatives of trigonometric functions:

   (a) True or False: If $f'(\theta) = -\sin(\theta)$, then $f(\theta) = \cos(\theta)$.

   (b) True or False: If θ is one of the non-right angles in a right triangle and $\sin(\theta) = \dfrac{2}{3}$, then the hypotenuse of the triangle must have length 3.

   (c) Differentiate both sides of the identity $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$ to obtain a new trigonometric identity.