Worksheet # 17: Linear Approximation and Applications

1. For each of the following, use a linear approximation to the change in the function and a convenient nearby point to estimate the value:
   (a) $(3.01)^3$
   (b) $\sqrt{17}$
   (c) $8.06^{2/3}$
   (d) $\tan(44°)$

2. What is the relation between the linearization of a function $f(x)$ at $x = a$ and the tangent line to the graph of the function $f(x)$ at $x = a$ on the graph?

3. Use the linearization of $\sqrt{x}$ at $x = 16$ to estimate $\sqrt{18}$;
   (a) Find a decimal approximation to $\sqrt{18}$ using a calculator.
   (b) Compute both the error and the percentage error.

4. Suppose we want to paint a sphere of radius 200 cm with a coat of paint .2 cm thick. Use a linear approximation to approximate the amount of paint we need to do the job.

5. Let $f(x) = \sqrt{16 + x}$. First, find the linearization to $f(x)$ at $x = 0$, then use the linearization to estimate $\sqrt{15.75}$. Present your solution as a rational number.

6. Find the linearization $L(x)$ to the function $f(x) = \sqrt{1 - 2x}$ at $x = -4$.

7. Find the linearization $L(x)$ to the function $f(x) = \sqrt[3]{x + 4}$ at $x = 4$, then use the linearization to estimate $\sqrt[3]{8.25}$.

8. Your physics professor tells you that you can replace $\sin(\theta)$ with θ when θ is close to zero. Explain why this is reasonable.

9. Suppose we measure the radius of a sphere as 10 cm with an accuracy of $\pm$ .5 cm. Use linear approximations to estimate the maximum error in:
   (a) the computed surface area.
   (b) the computed volume.

10. Suppose that $y = y(x)$ is a differentiable function which is defined near $x = 2$, satisfies $y(2) = -1$ and

$$x^2 + 3xy^2 + y^3 = 9.$$

Use the linear approximation to the change in y to approximate the value of $y(1.91)$.