

Worksheet # 20: L'Hôpital's Rule & Optimization

1. Carefully state l'Hôpital's Rule.
2. Compute the following limits. Use l'Hôpital's Rule where appropriate but first check that no easier method will solve the problem.

(a) $\lim_{x \rightarrow 1} \frac{x^9 - 1}{x^5 - 1}$

(c) $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2}$

(b) $\lim_{x \rightarrow 0} \frac{\sin(4x)}{\tan(5x)}$

(d) $\lim_{x \rightarrow 1} \frac{x^2 + 2x - 2}{x^2 - 2x + 2}$

3. Find the dimensions of x and y of the rectangle of maximum area that can be formed using 3 meters of wire.
 - (a) What is the constraint equation relating x and y ?
 - (b) Find a formula for the area in terms of x alone.
 - (c) Solve the optimization problem.
4. A flexible tube of length 4 m is bent into an L-shape. Where should the bend be made to minimize the distance between the two ends?
5. A rancher will use 600 m of fencing to build a corral in the shape of a semicircle on top of a rectangle. Find the dimensions that maximize the area of the corral. (Hint: draw a picture)
6. Find the value A for which we can use l'Hôpital's rule to evaluate the limit

$$\lim_{x \rightarrow 2} \frac{x^2 + Ax - 2}{x - 2}.$$

For this value of A , give the value of the limit.

7. Compute the following limits. Use l'Hôpital's Rule where appropriate but first check that no easier method will solve the problem.

(a) $\lim_{x \rightarrow -\infty} x^2 e^x$

(c) $\lim_{x \rightarrow \pi} \frac{\cos(x) + 1}{x^2 - \pi^2}$

(b) $\lim_{x \rightarrow \infty} x^3 e^{-x^2}$

(d) $\lim_{x \rightarrow \infty} x \cdot \left(\arctan(x) - \frac{\pi}{2} \right)$

8. Find the dimensions x and y of the rectangle inscribed in a circle of radius r that maximizes the quantity xy^2 .
9. Find the point on the line $y = x$ closest to the point $(1, 0)$. Find the point on the line $y = x$ closest to the point $(r, 1 - r)$. What do these points look like graphically?